# Photon counting interferometry to detect geontropic space-time fluctuations with GQuEST

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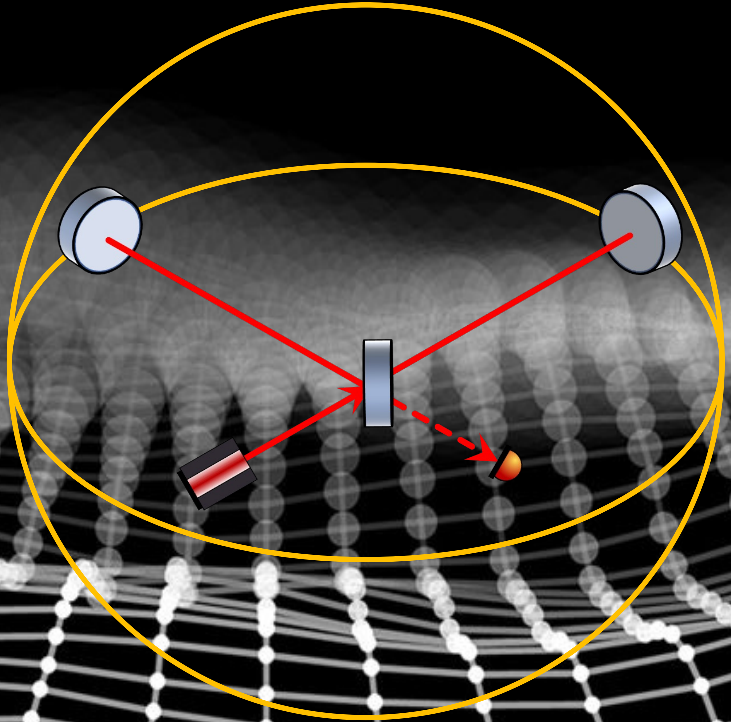


# Outline

1. Introduction to GQuEST
2. Holographic Quantum Gravity Fluctuations
3. Basics of Laser Interferometry
4. Homodyne Readout
5. Photon Counting Readout
6. Classical Noises

# GQuEST in short

- Twin lab-scale (5 m) Michelson laser interferometers

- Search for holographic quantum space-time fluctuations

  i.e. Gravity from the Quantum Entanglement of Space-Time

- Novel ‘*photon counting readout*’ evades quantum noise

- Subsystems are under construction, awaiting new lab completion

# Why use an interferometer to detect quantum gravity?

Gravity is Geometry

Gravity is Quantum-Mechanical

→ Geometry is Quantum-Mechanical

→ Distance measurements exhibit quantum fluctuations

# LIGO vs. GQuEST

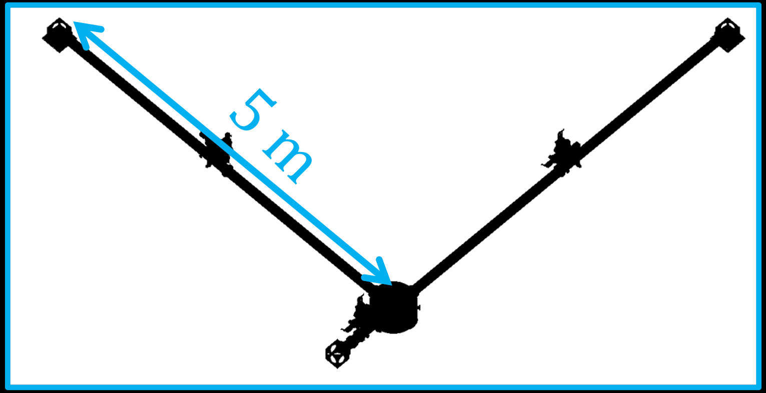
## Sensitivity to length changes:

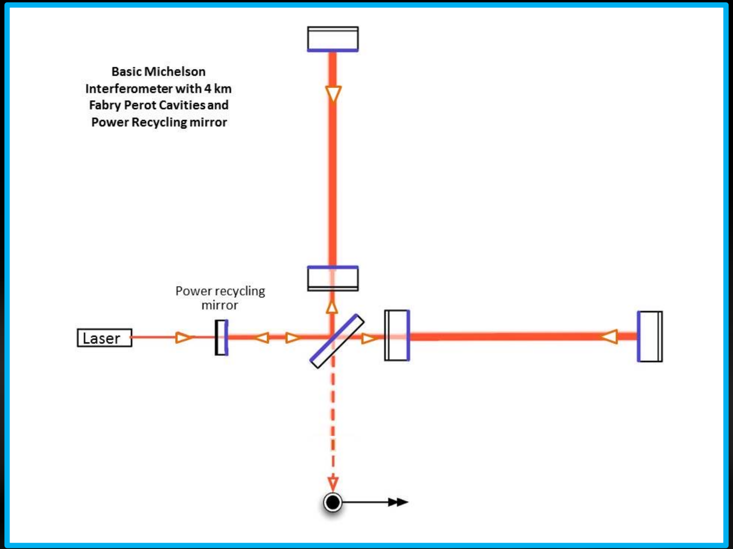

$$\delta L \approx 10^{-20}\ \mathrm{m}$$

@ 100 Hz

$$\delta L \approx 10^{-21}\ \mathrm{m}$$

@ 10 MHz

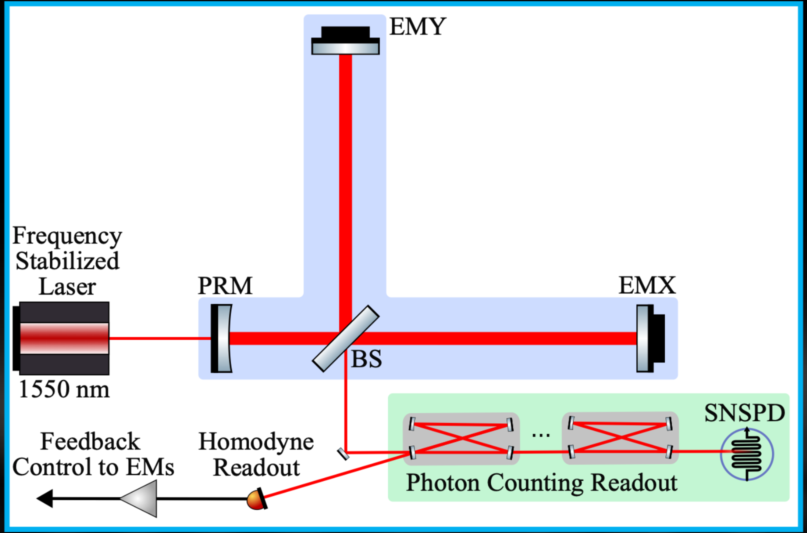



## Detection statistic:

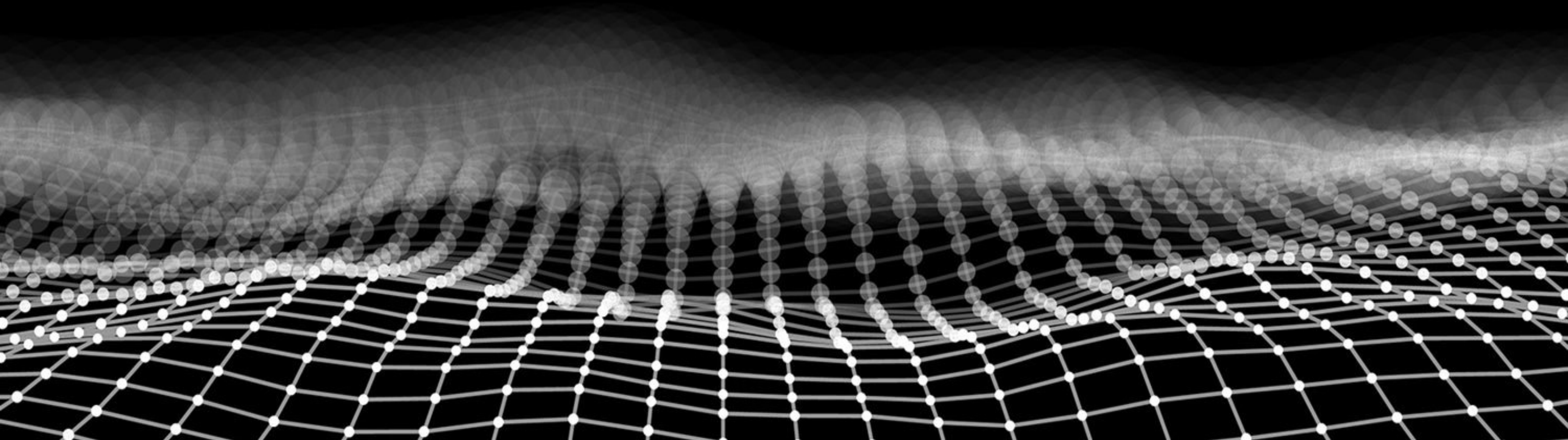
$$\propto \left[\frac{S_{\mathrm{Signal}}}{S_{\mathrm{Noise}}}\right]^2$$

$$\propto \frac{S_{\mathrm{Signal}}}{S_{\mathrm{Noise}}}$$




$$\frac{S_{\mathrm{signal}}}{S_{\mathrm{noise}}} \ll 1$$

# Heuristic Holographic Quantum Gravity



# Heuristics of Holographic Quantum Gravity

- Covariant Entropy Bound:

Area of causal horizon

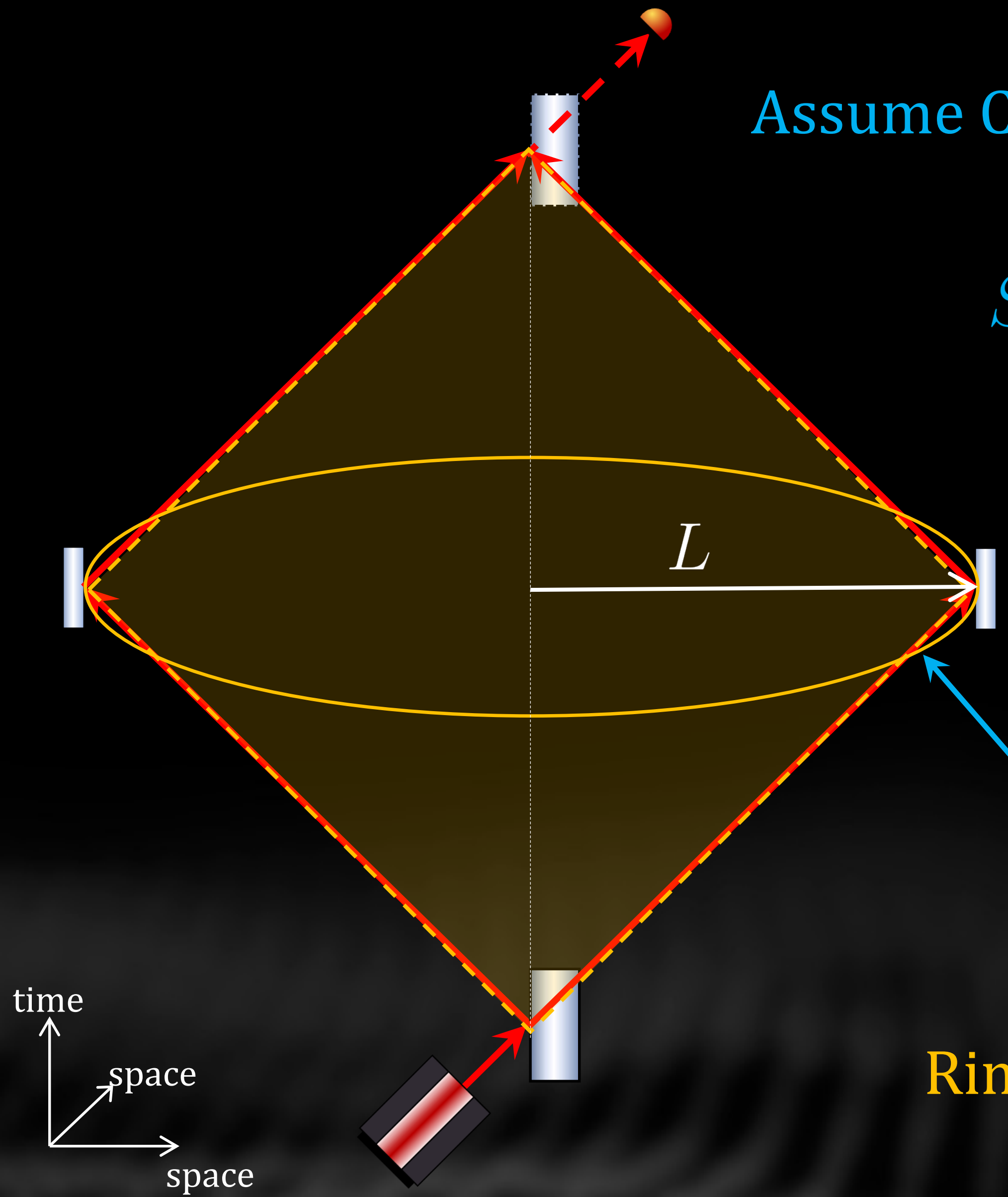
$$\boxed{S} \leq \frac{\boxed{A_H}}{4G} = \frac{8\pi^2 L^2}{(l_{\mathrm{P}})^2}$$

Radius of spherical horizon

Entropy ‘inside’ horizon

→ Saturated for black holes (Generalised 2nd law of T.D.)

# Heuristics of Holographic Quantum Gravity

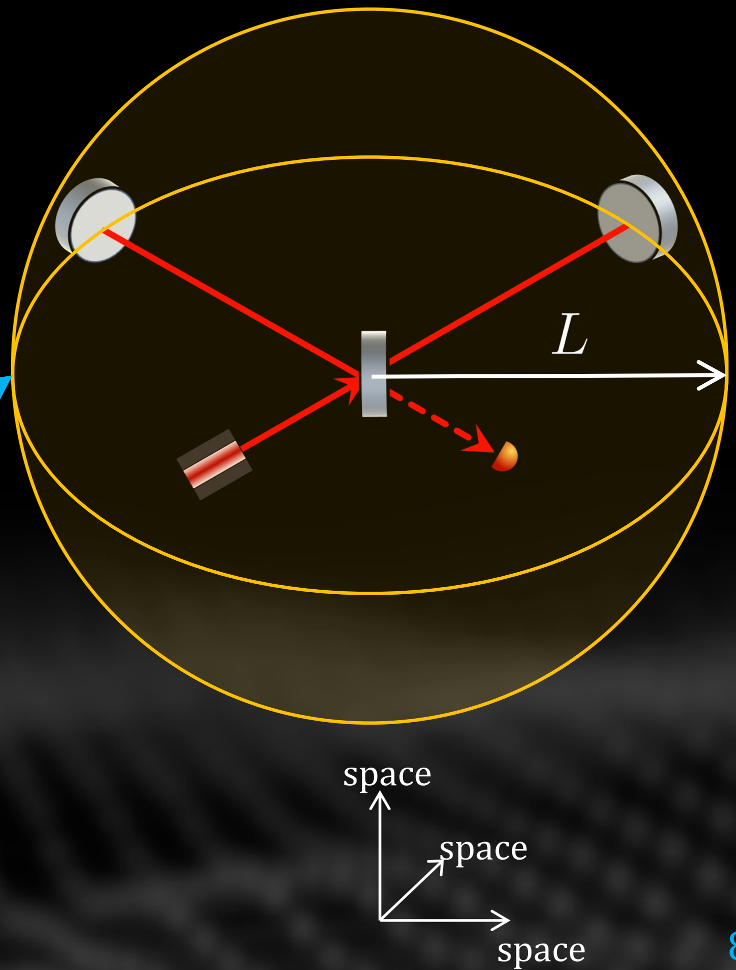



Assume Covariant Entropy Bound is saturated:

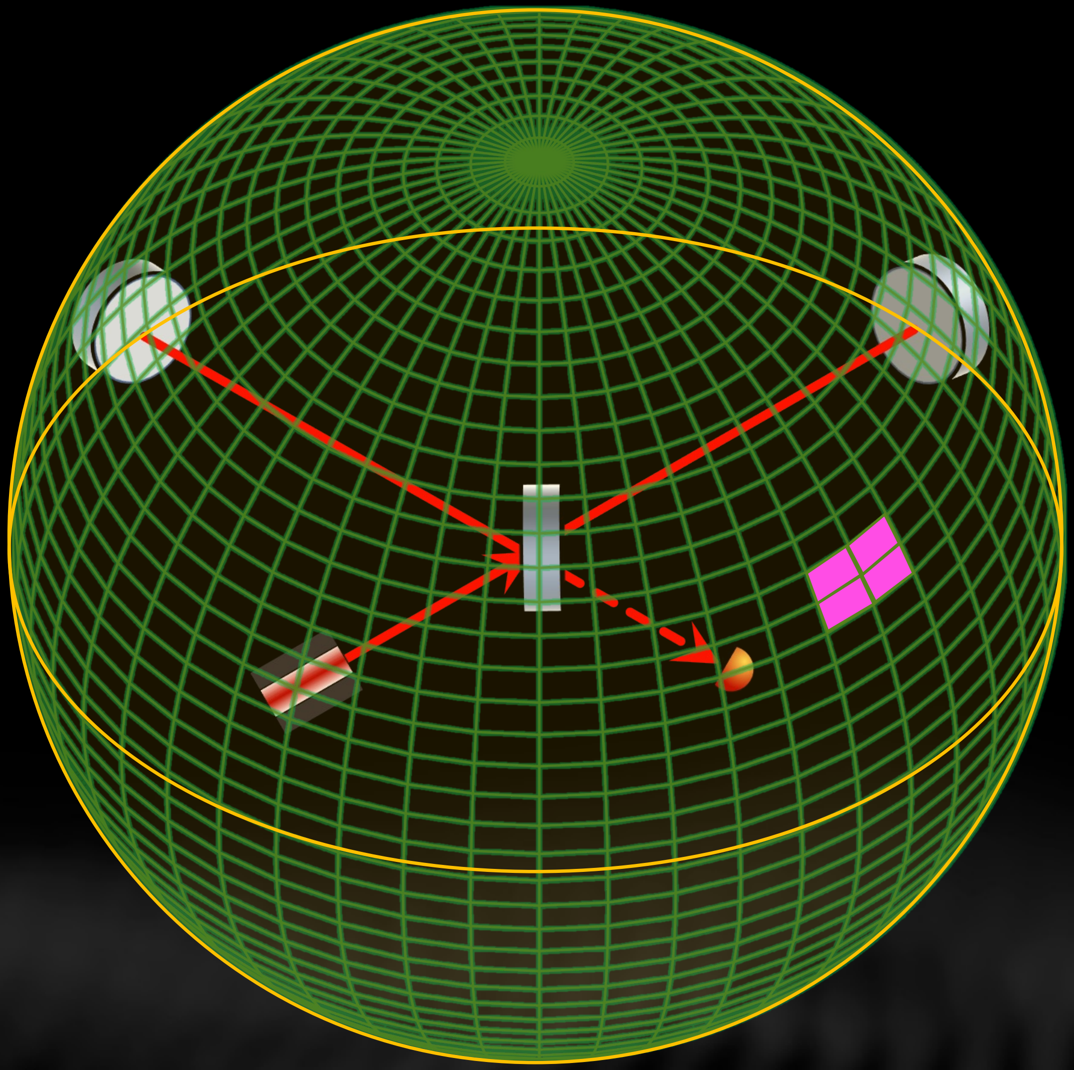
$$S = \frac{A_H}{4G} = \frac{8\pi^2 L^2}{(l_{\mathrm{P}})^2}$$

$$\boxed{S \propto \frac{L^2}{(l_{\mathrm{P}})^2}}$$

Rindler/Killing horizon



# Heuristics of Holographic Quantum Gravity



Associate degrees of freedom to the entropy:

$$\boxed{N_{\text{d.o.f.}} \propto S}$$

Assume degrees of freedom undergo quantum fluctuations:

$$\boxed{\frac{\delta E}{E} \propto \frac{\sqrt{N_{\text{d.o.f.}}}}{N_{\text{d.o.f.}}}}$$

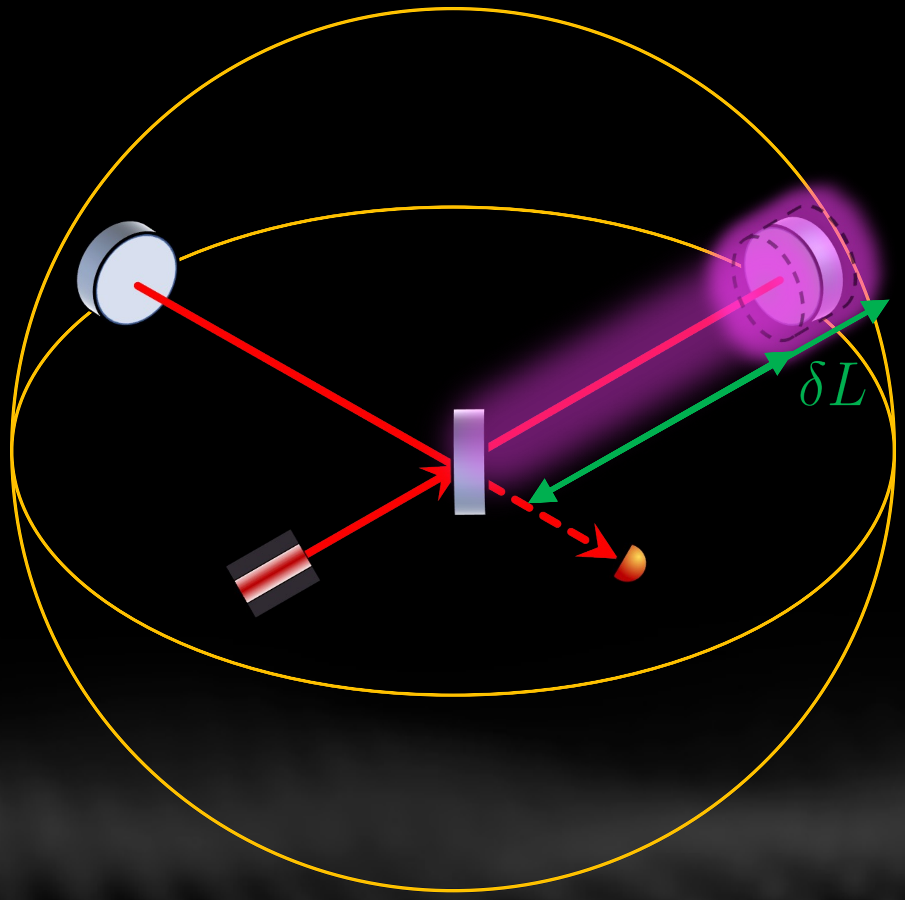
# Heuristics of Holographic Quantum Gravity

Gravity QM Stat. Mech. Holography

$$\left(\frac{\delta L}{L}\right)^2 \propto \frac{\delta E}{E} \propto \frac{\sqrt{N_{\mathrm{d.o.f.}}}}{N_{\mathrm{d.o.f.}}} \propto \frac{\sqrt{S}}{S} \propto \frac{l_{\mathrm{P}}}{L}$$

$$\rightarrow \boxed{\delta L \propto \sqrt{l_{\mathrm{P}}\, L}}$$

$$= \mathcal{O}(10^{-18})\ \mathrm{m}$$

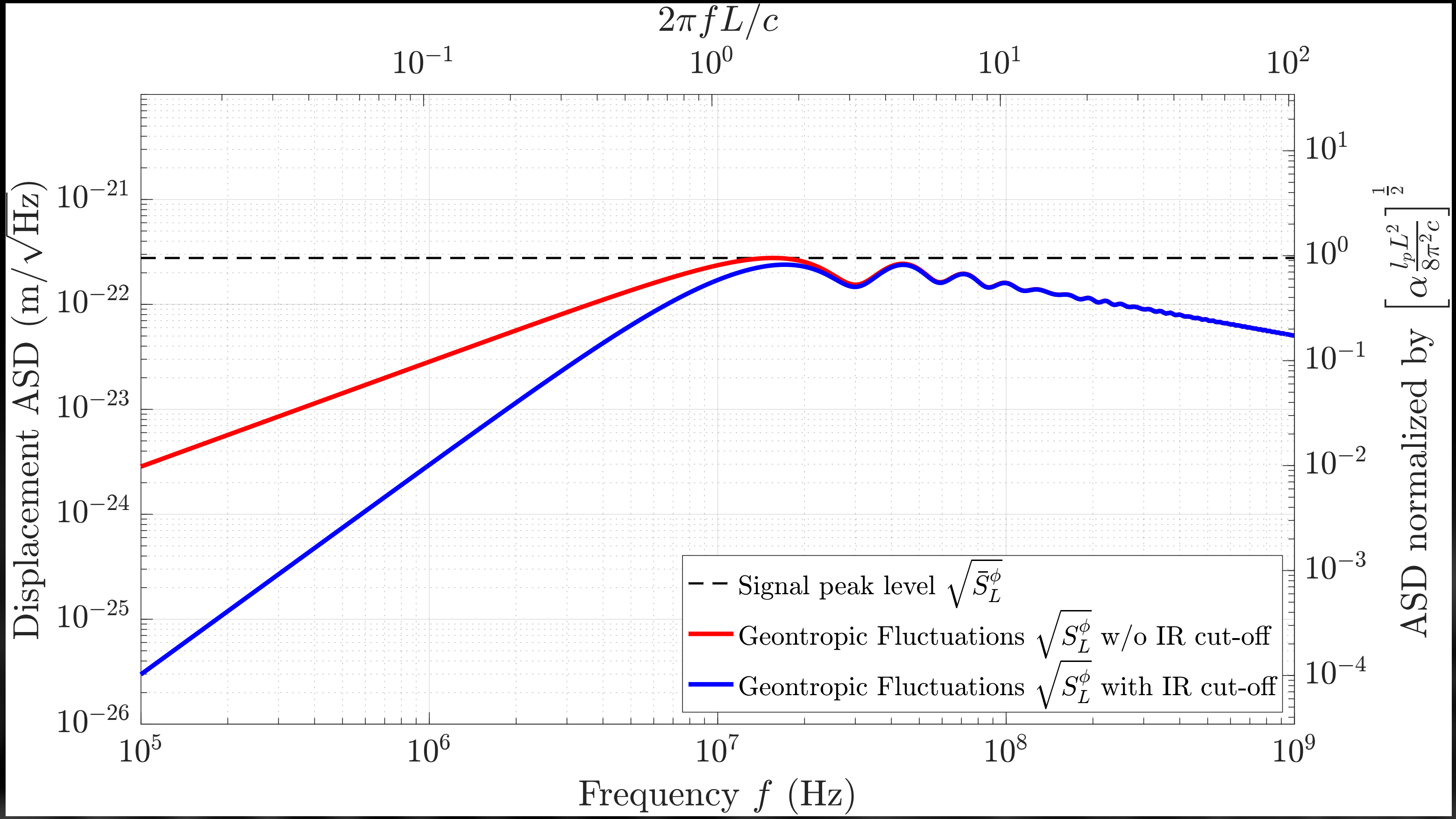


# Space-time fluctuations: Pixellon Model

- Associate a stochastic scalar field to holographic degrees of freedom: $\phi(\vec{x}, t)$
- The field gravitates, perturbing the metric: $ds^2 = -dt^2 + (1-\phi)\left(dr^2 + r^2 d\Omega^2\right)$

→IFO signal spectrum:

$$\sqrt{S^{\mathrm{sig}}}$$



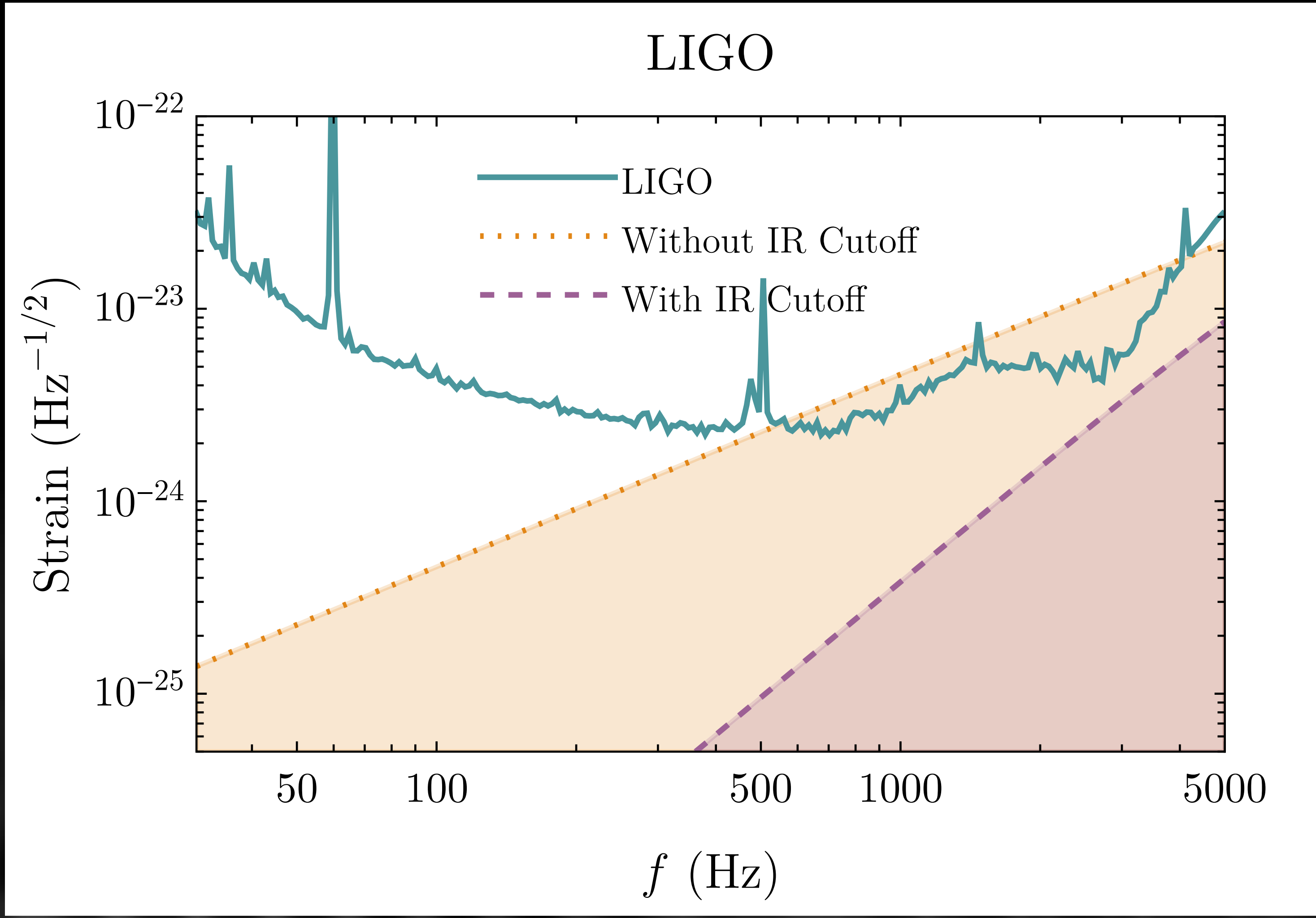
# Pixellon Model: LIGO constraints

$$\max\left(\sqrt{S_L^{\text{sig}}(f)}\right) = \sqrt{\alpha\,\frac{l_p L^2}{8\pi^2 c}}$$

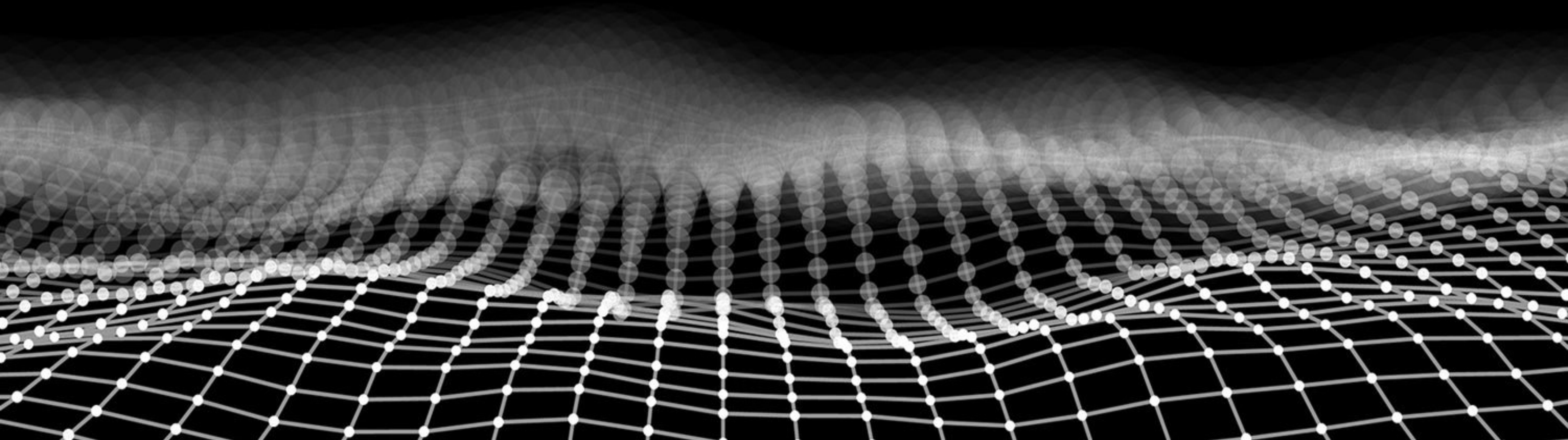
'Low'-frequency LIGO constraints:

$\alpha \lesssim 3$ (with IR cut-off)

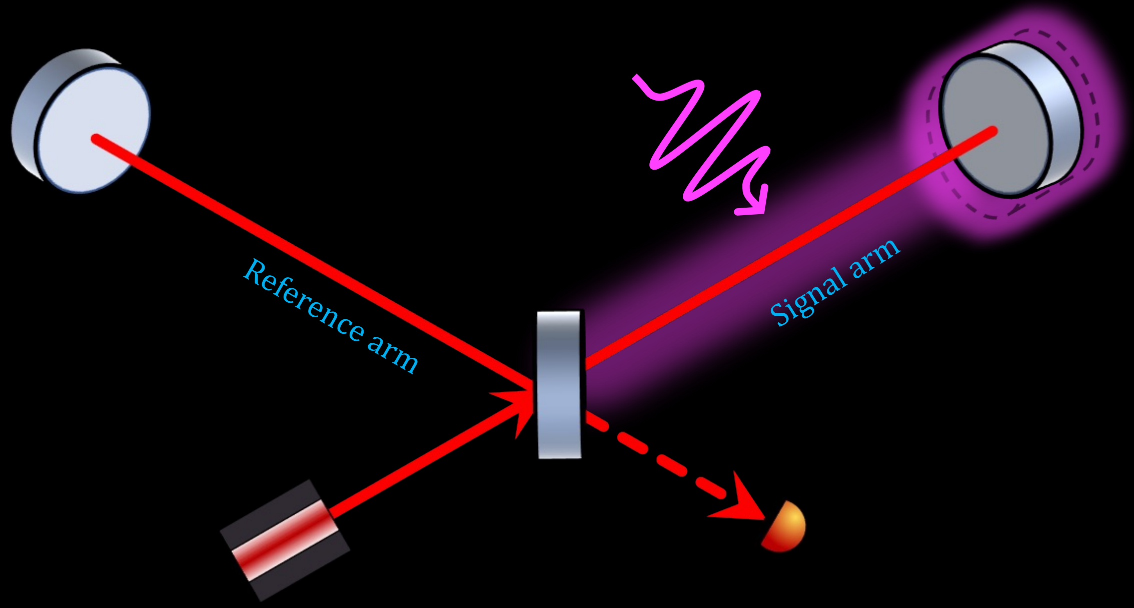
$\alpha \lesssim 0.1$ (without IR cut-off)



# Laser Interferometry & Photon Counting



# Laser interferometry: measuring phase modulations




Perturbations $\Longrightarrow$ $\boxed{\delta L \quad \text{or} \quad \delta\phi}$

Modulates the power at the output

Phase modulation of the carrier field:

$$E_\mathrm{c}(t) = |E_\mathrm{c}|e^{i\omega_\mathrm{c}t} \longrightarrow E(t) = \underbrace{|E_\mathrm{c}|e^{i(\omega_\mathrm{c}t+\boxed{\delta\phi(t)})}}_{\text{Field in signal arm}}$$

# Sideband fields

$$E(t) = |E_\mathrm{c}|e^{i(\omega_\mathrm{c}t+\delta\phi(t))}$$

$$\boxed{\delta\phi(t) = \Phi\sin(\Omega t)}$$

Expansion for $\Phi \ll 1$

$$E(t) = |E_\mathrm{c}|e^{i(\omega_\mathrm{c}t + \Phi\sin(\Omega t))} \approx |E_\mathrm{c}|e^{i\omega_\mathrm{c}t}\left(1 + \frac{i}{2}\Phi\left[e^{-i\Omega t} + e^{+i\Omega t}\right]\right)$$

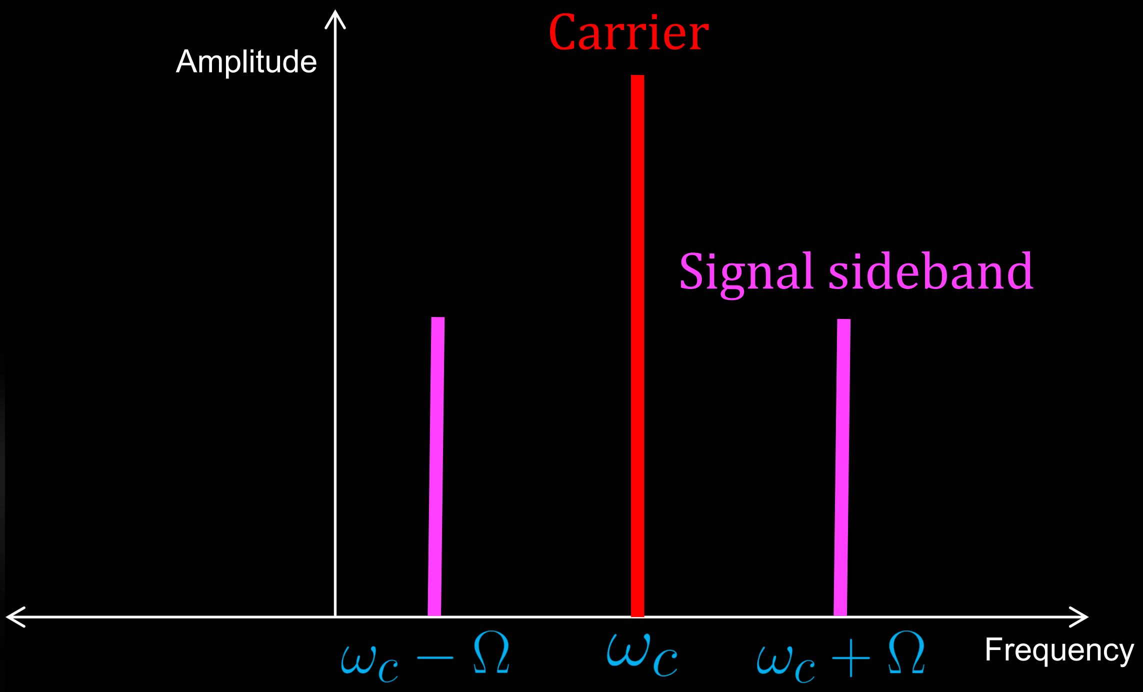
$$\rightarrow E(t) \approx E_\mathrm{c} + E_\mathrm{sig}$$

Carrier field:

$$E_\mathrm{c} = |E_\mathrm{c}|e^{i\omega_\mathrm{c}t}$$

+ Signal sideband fields

$$E_\mathrm{sig} = \frac{1}{2}|E_\mathrm{c}|\Phi\left[e^{i(\omega_\mathrm{c}-\Omega)t} + e^{i(\omega_\mathrm{c}+\Omega)t}\right]$$



# Sideband fields

$$E(t) = |E_c| e^{i(\omega_c t + \delta\phi(t))}$$

$$\delta\phi(t) \equiv \boxed{A_{\text{sig}}(t)} \sim \mathcal{F}^{-1}\left\{\sqrt{S_\phi^{\text{sig}}(f)}\right\} e^{i\eta_{\text{rand.}}(t)}$$

Expand for $A_{\text{sig}} \ll 1$

$$E(t) = |E_c| e^{i(\omega_c t + A_{\text{sig}}(t))}$$
$$\approx |E_c| e^{i\omega_c t}\left(1 + iA_{\text{sig}}\right)$$

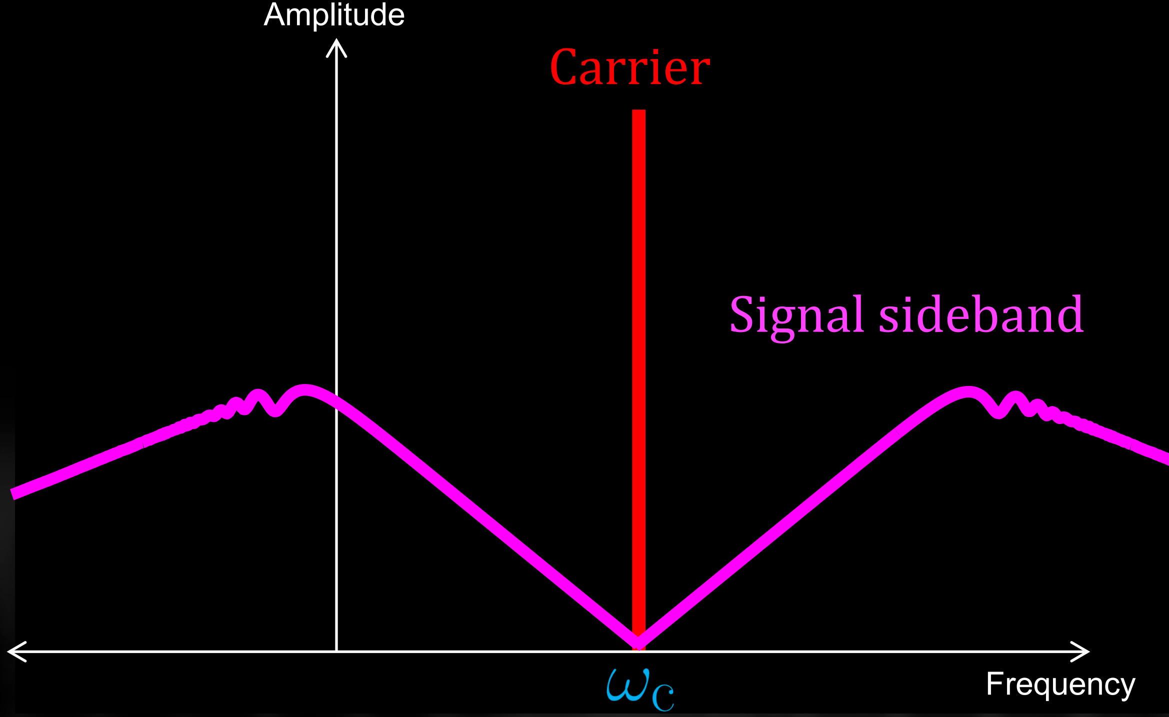
→ $E(t) \approx E_c + E_{\text{sig}}$

Carrier field:

$$E_c = |E_c| e^{i\omega_c t}$$

+ Signal sideband fields

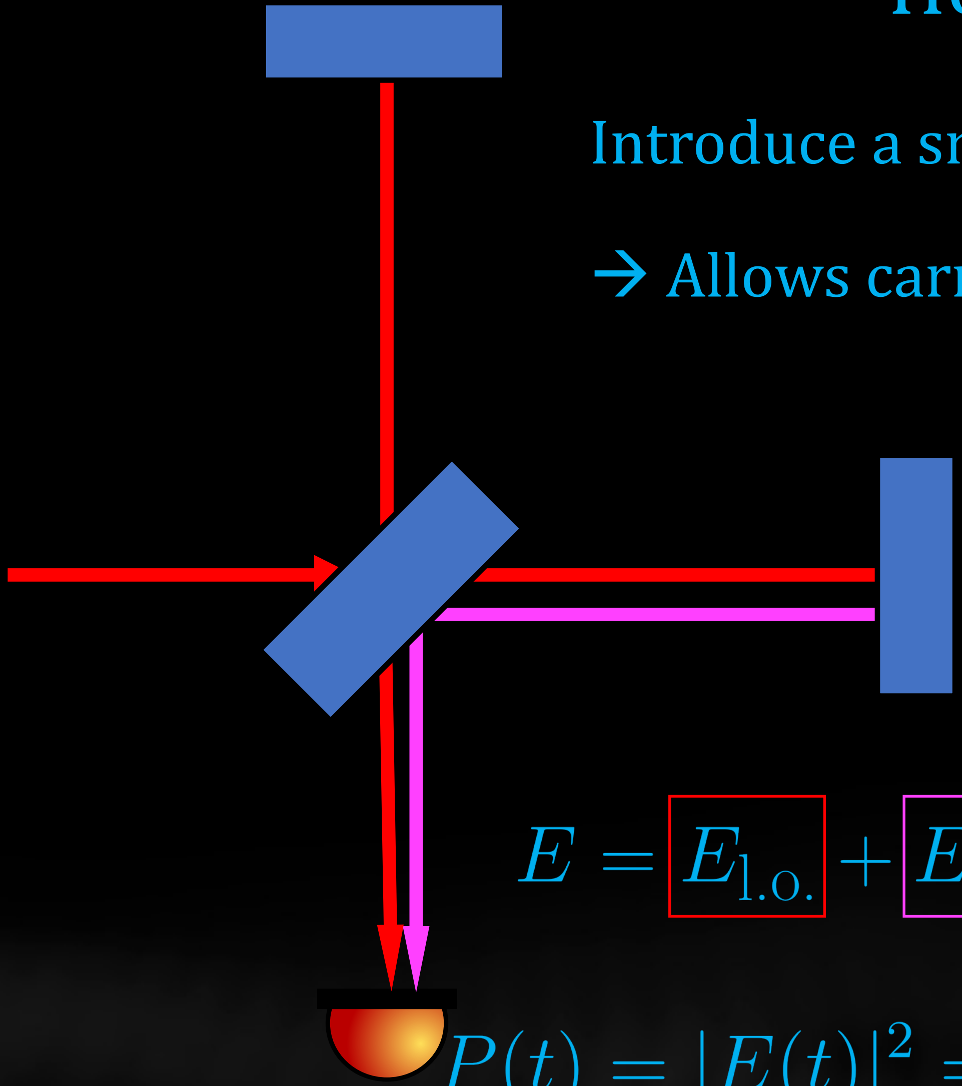
$$E_{\text{sig}} = E_c A_{\text{sig}}$$

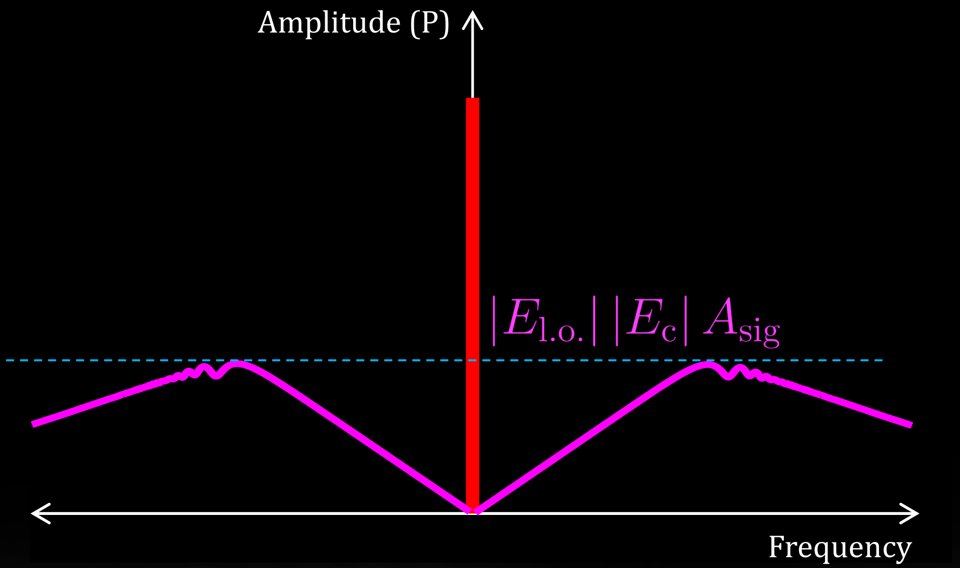


# Homodyne Readout

Introduce a small static arm-length difference

→ Allows carrier field to leak into the output: $E_{\mathrm{l.o.}} = |E_{\mathrm{l.o.}}| e^{i\omega_{\mathrm{c}} t}$






$$E = \boxed{E_{\mathrm{l.o.}}} + \boxed{E_{\mathrm{sig}}}$$

$$P(t) = |E(t)|^2 = \boxed{|E_{\mathrm{l.o.}}|^2} + \underbrace{\boxed{2|E_{\mathrm{l.o.}}||E_{\mathrm{c}}|A_{\mathrm{sig}}}}_{\text{Sidebands beat with ‘local oscillator’}} + \boxed{|E_{\mathrm{c}}|^2 A_{\mathrm{sig}}^2}$$

# Quantum Shot Noise

$$E = \sqrt{P}$$

Heisenberg uncertainty for coherent optical state:

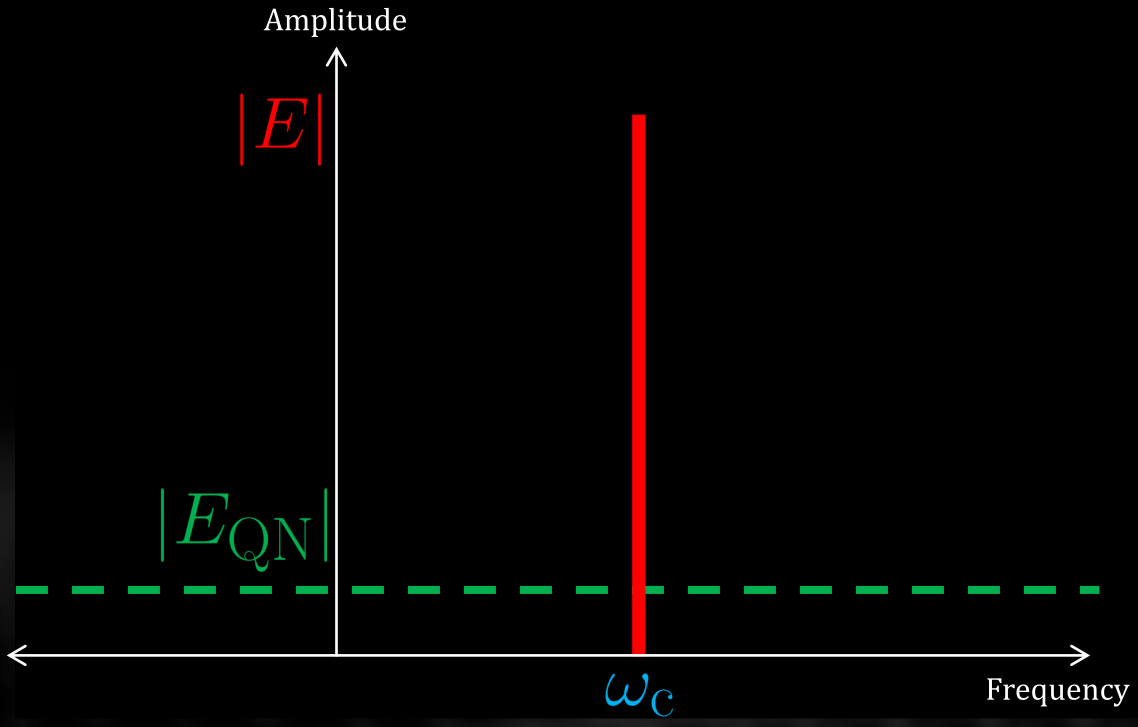
$$\left.\begin{aligned} \sigma_N \cdot \sigma_\phi &= 1 \\ \sigma_N = \sqrt{N} &= \sqrt{\frac{|E|^2}{\hbar\omega}} \end{aligned}\right\} \sigma_\phi = \sqrt{\frac{\hbar\omega}{|E|^2}} = \sqrt{2S_\phi^{\mathrm{QN}}}$$

$(t_{\mathrm{meas.}} = 1\ \mathrm{s})$

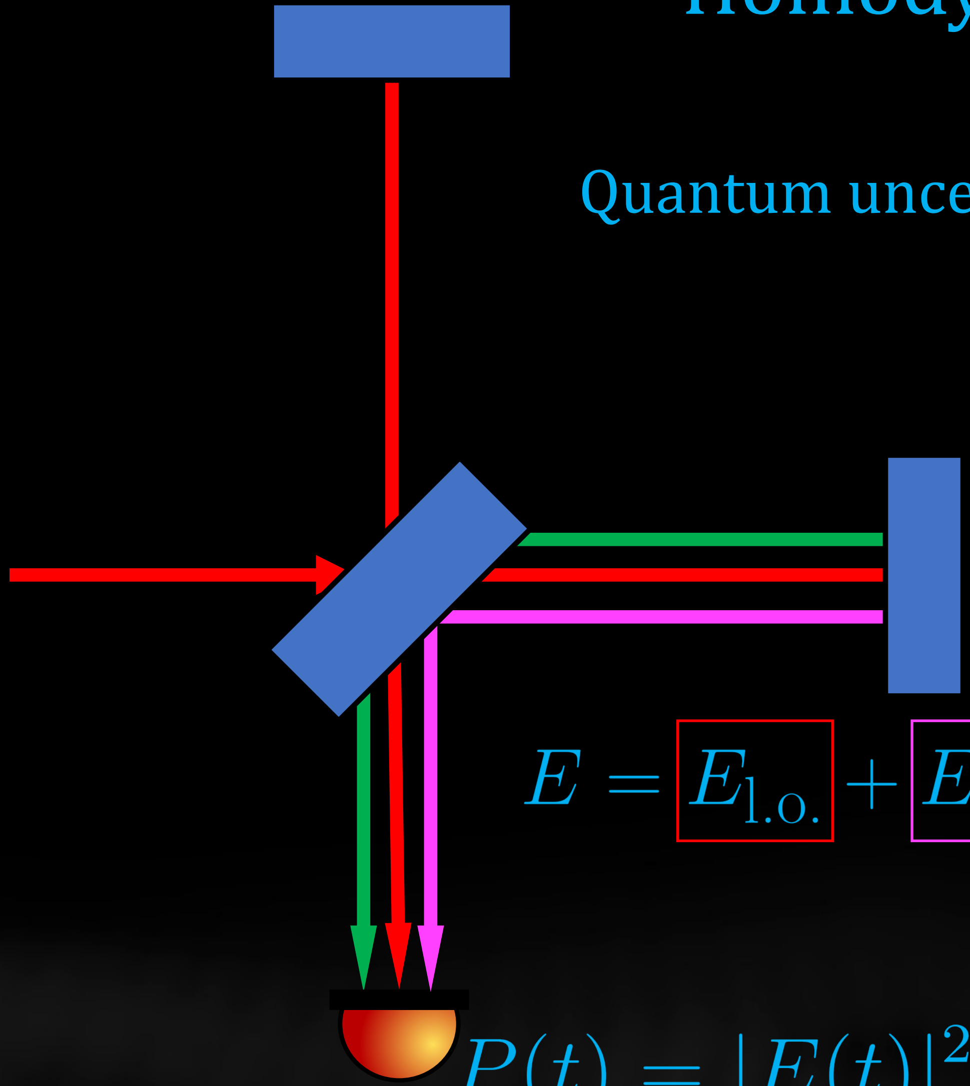
$$E(t) = |E_\mathrm{c}|e^{i\omega_\mathrm{c}t} \boxed{+\sigma_\phi}$$

Quantum noise 'sidebands':

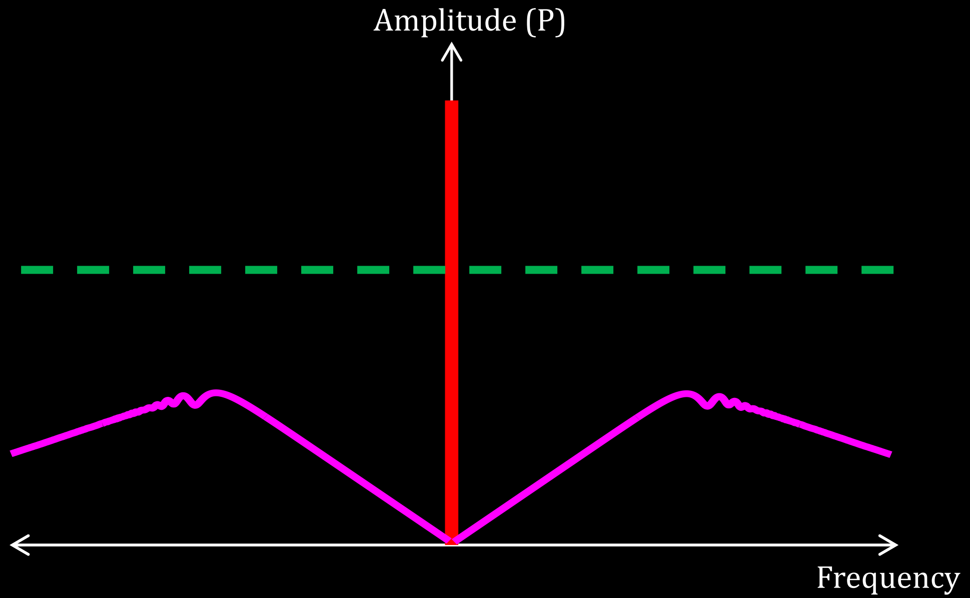
$$\boxed{E_{\mathrm{QN}} = \sqrt{\frac{\hbar\omega_c}{2}} \sum_{\omega=-\infty}^{\infty} \left[\hat{a}^\dagger e^{i(\omega_\mathrm{c}+\omega)t} + \hat{a} e^{i(\omega_\mathrm{c}-\omega)t}\right]}$$



# Homodyne Readout: Shot Noise



Quantum uncertainty produces measured shot noise




$$E = \boxed{E_{\text{l.o.}}} + \boxed{E_{\text{sig}}} + \boxed{E_{\text{QN}}}$$

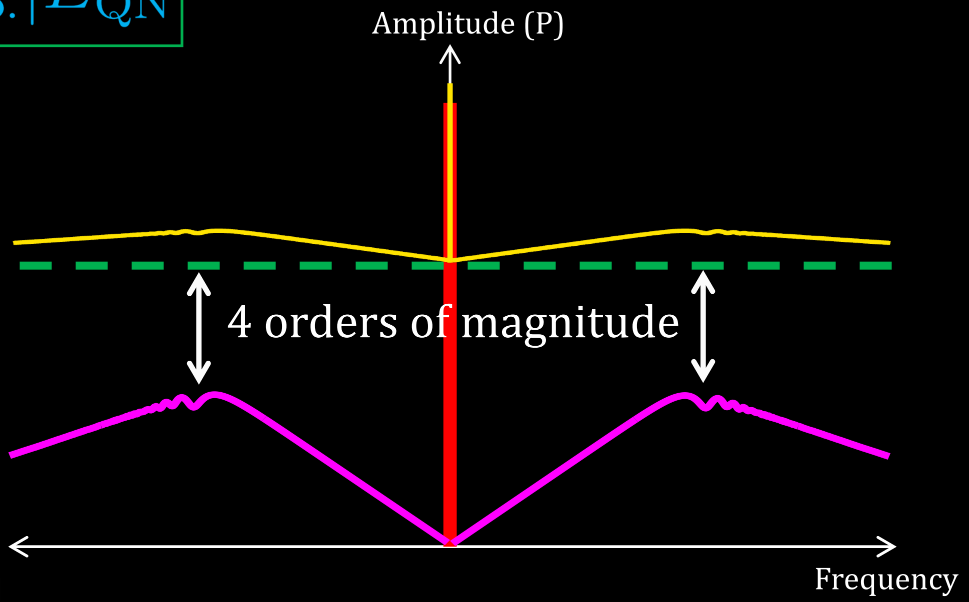
$$P(t) = |E(t)|^2 = \boxed{|E_{\text{l.o.}}|^2} + \boxed{2|E_{\text{l.o.}}||E_{\text{c}}|A_{\text{sig}}} + \boxed{2|E_{\text{l.o.}}|E_{\text{QN}}}$$

$$+ \text{ smaller terms}$$

# Homodyne Readout: Statistics

$$P(t) = \boxed{|E(t)|^2} = \boxed{|E_{\mathrm{l.o.}}|^2} + \boxed{2|E_{\mathrm{l.o.}}||E_{\mathrm{c}}|A_{\mathrm{sig}}} + \boxed{2|E_{\mathrm{l.o.}}|E_{\mathrm{QN}}}$$

$$+ \,\text{smaller terms}$$




## Detection statistic:

$$\chi^2 = \int \frac{\left(S_{\mathrm{meas}}(f) - S_{\mathrm{noise}}\right)^2}{\mathrm{Var}\left(S_{\mathrm{meas}}\right)} df \quad \propto \left(\frac{S_{\mathrm{signal}}}{S_{\mathrm{noise}}}\right)^2$$

Can we do better?

→Yes, with photon counting!

# Photon Counting: Intuition

- Homodyne readout measures time-dependence, i.e. phase/frequency of the signal

- The signal model does not specify these properties…

→ time-dependence/phase/frequency info is useless for finding a signal that is stationary/stochastic/broadband

→ Devise a quantum measurement that does not provide useless info, in exchange for useful info

# Photon Counting

Measure the number of photons exactly:

$$\left.\begin{aligned} \sigma_N &= 0 \\ \sigma_N \cdot \sigma_\phi &= 1 \end{aligned}\right\} \implies \sigma_\phi = \infty$$

→No phase info measured

→Maximum info on signal power

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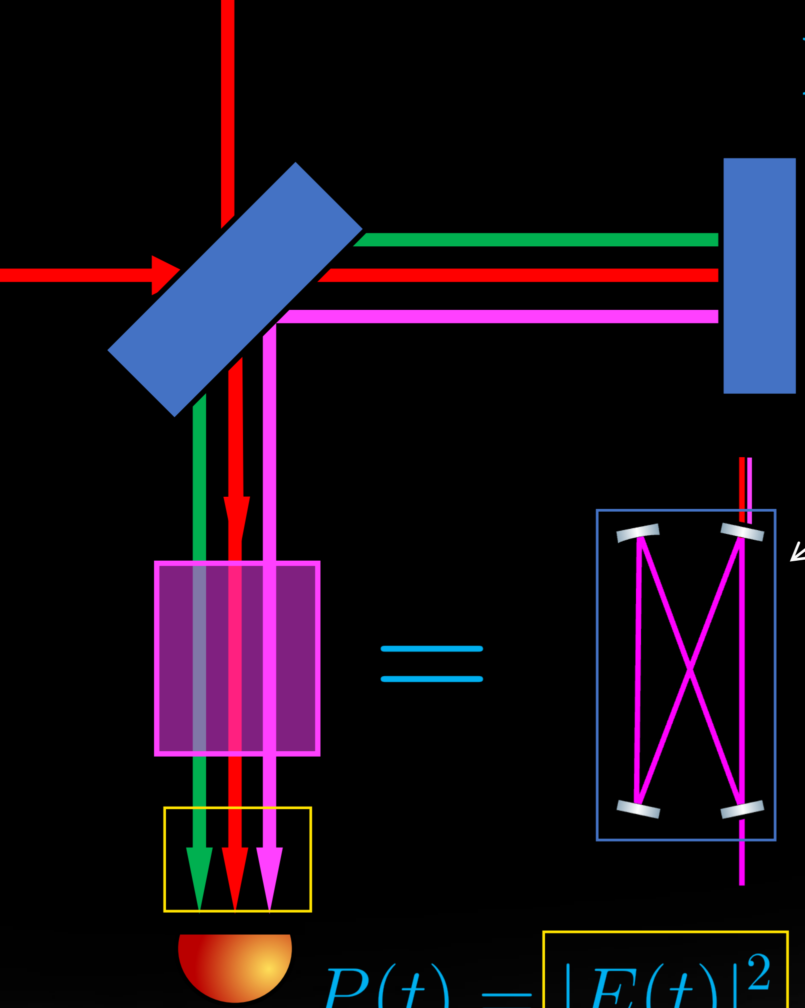
Practical challenges with this approach:

- Too much light to discern single signal photons
- Too many non-signal photons

→ Can't count photons precisely

$$\sigma_N \neq 0$$

# Photon Counting: Filtering



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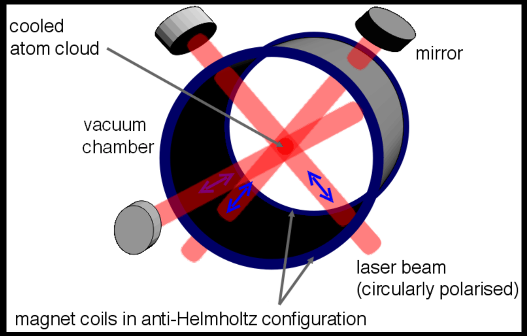
Narrowband optical filter

or




or

?

$$P(t) = \boxed{|E(t)|^2}$$

$$= \cancel{\underbrace{|E_{\mathrm{l.o.}}|}_{\to\, 0}\left(|E_{\mathrm{l.o.}}| + |E_{\mathrm{c}}|A_{\mathrm{sig}} + E_{\mathrm{QN}}\right) +} \boxed{|E_{\mathrm{c}}|^2 A_{\mathrm{sig}}^2} + \boxed{2|E_{\mathrm{c}}|A_{\mathrm{sig}}E_{\mathrm{QN}}}$$

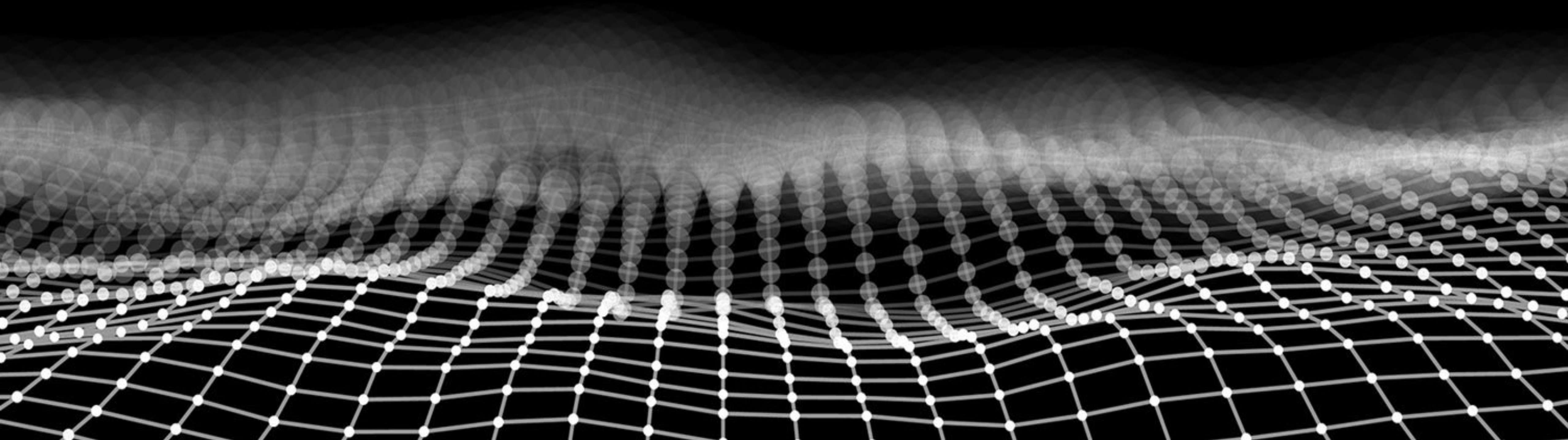
# Photon Counting: Statistics

## Detection statistic:

$$\left.\begin{aligned} \chi^2 &= \frac{N^2}{\mathrm{Var}\left(N\right)} = \frac{N^2}{N} = N \\ N &\propto \frac{\left| |E_\mathrm{c}|^2 A_\mathrm{sig}^2 \right|}{\hbar\omega} = \frac{S_\phi^\mathrm{sig}}{2S_\phi^{QN}} \end{aligned}\right\} \quad \boxed{\chi^2 \propto \frac{S_\mathrm{signal}}{S_\mathrm{noise}}}$$

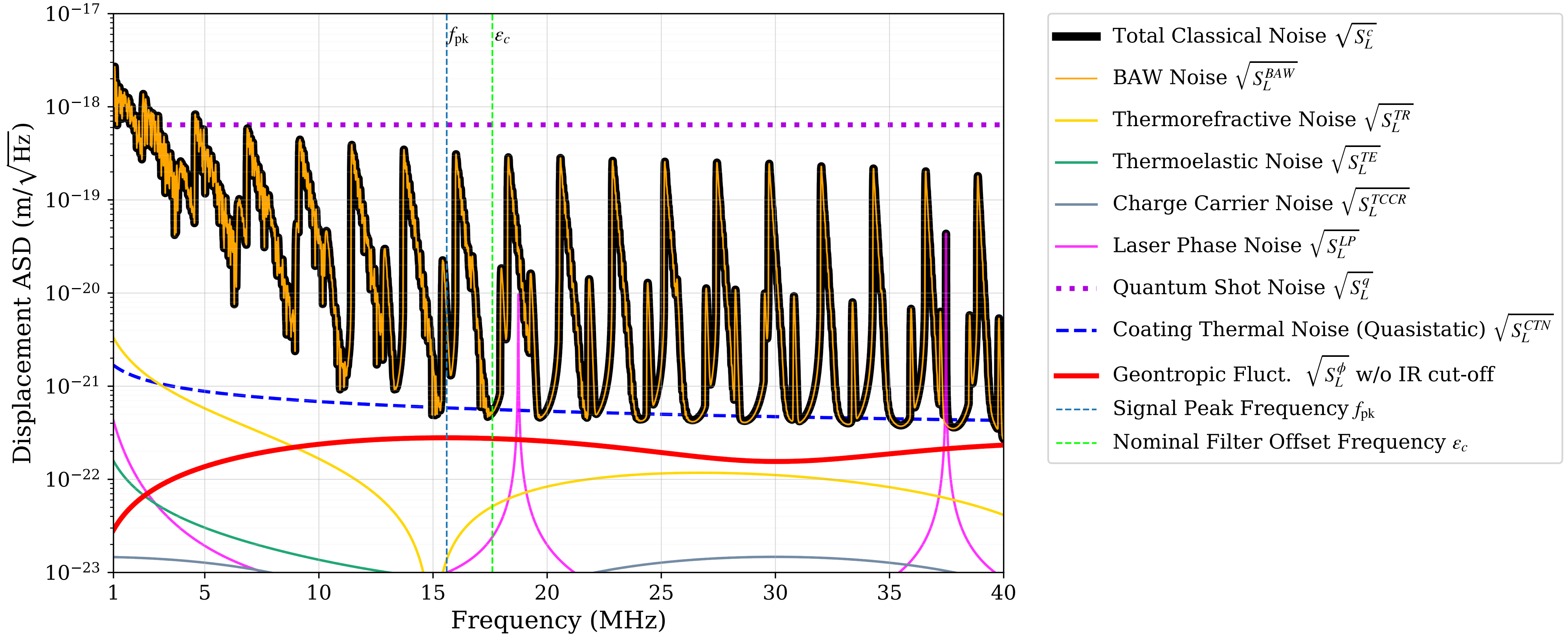
Recall for Homodyne readout: $\chi^2 \propto \left(\frac{S_\mathrm{signal}}{S_\mathrm{noise}}\right)^2$

# Classical Noise & Outlook



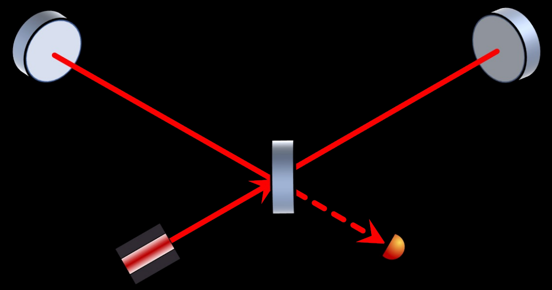
# Classical Noise

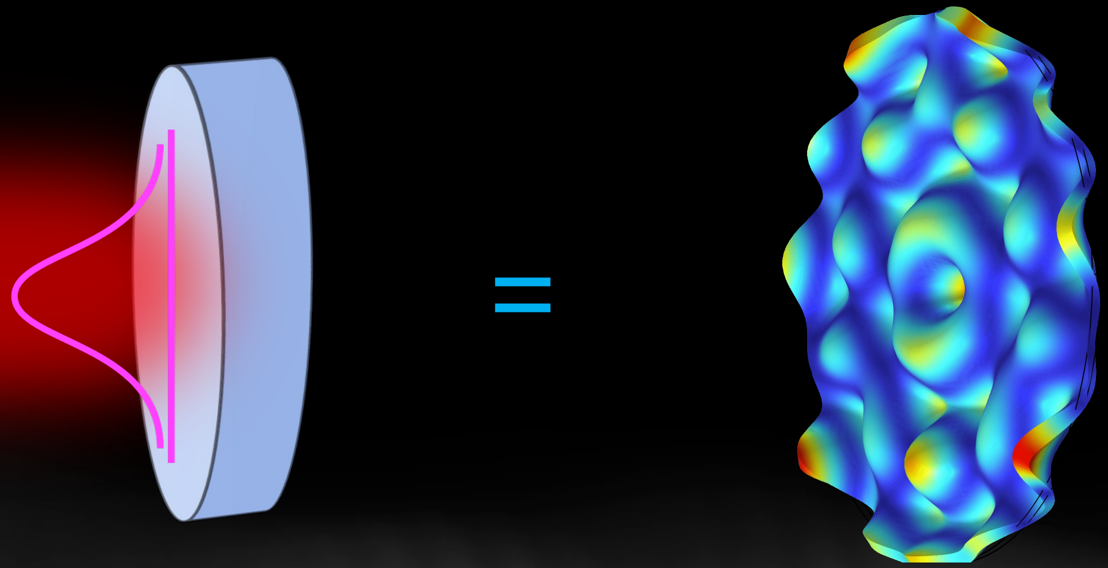
- Classical noise in the filter passband is indistinguishable from signal…



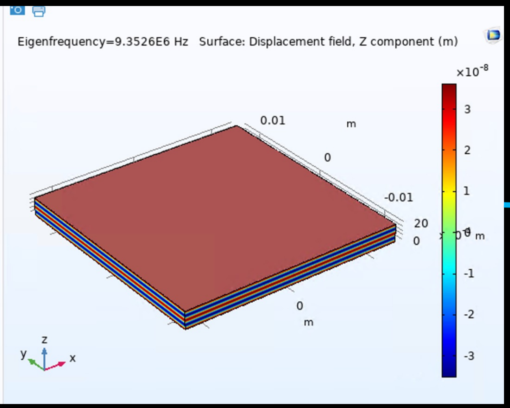

$$\chi^2 \propto \frac{\left(S_{\text{signal}}\right)^2}{\left(S^{\text{Qu.}}_{\text{noise}}\right)\left(S^{\text{Cl.}}_{\text{noise}}\right)}$$

# Classical Noise: Mirror Thermal Noise

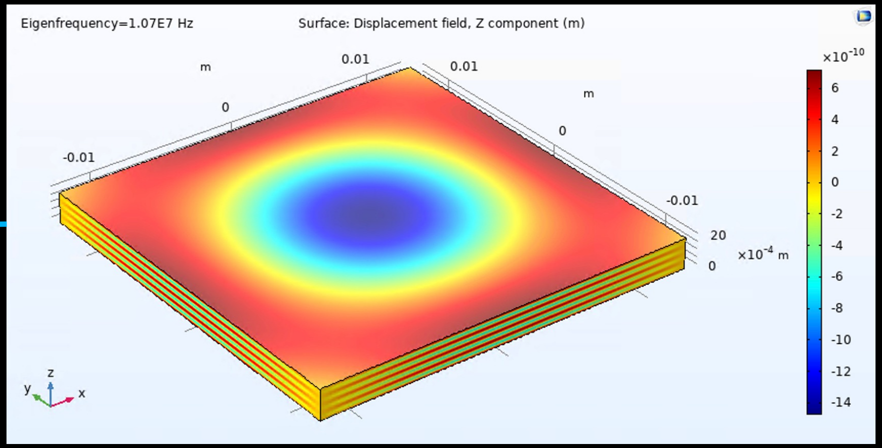




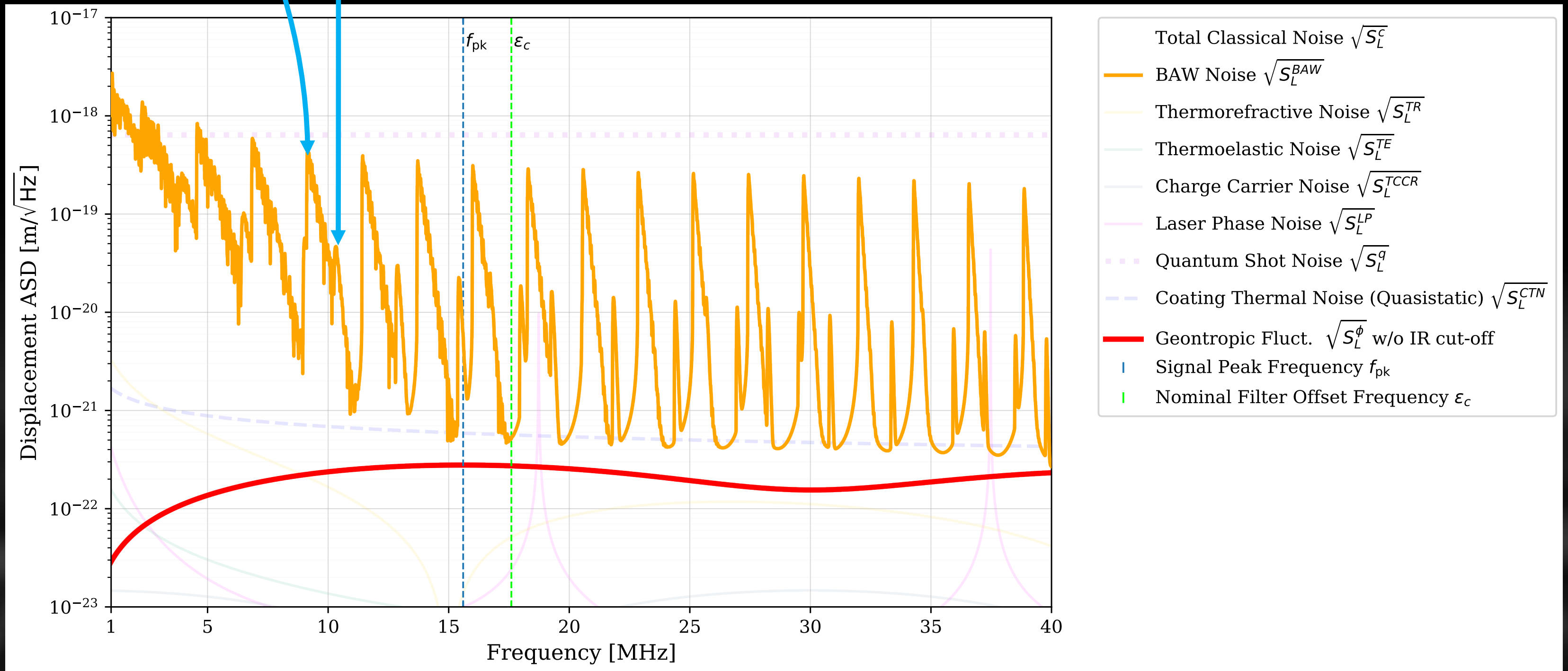
# Solid Normal Modes




Longitudinal

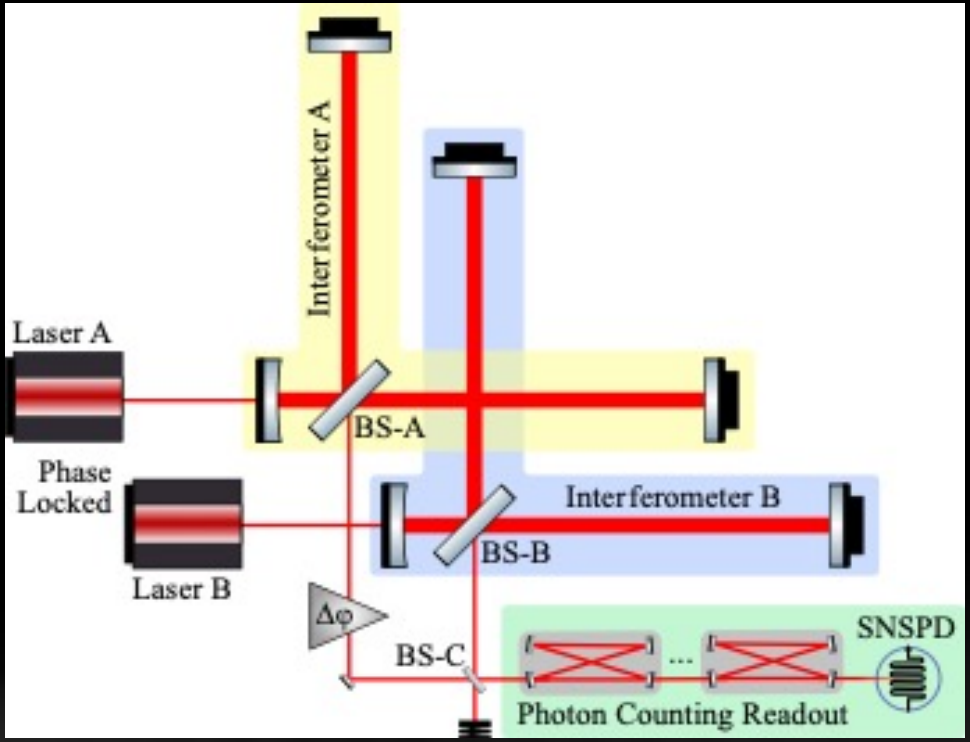



Transverse




Slide by Daniel Grass

# GQuEST: Outlook




$$\left(\frac{1}{\sigma}\right)^2 \approx \alpha^2 \left(\frac{T}{8.6 \cdot 10^3\ \mathrm{s}}\right) \left(\frac{P_{\mathrm{BS}}}{10\ \mathrm{kW}}\right) \left(\frac{L}{5\ \mathrm{m}}\right)^4 \left(\frac{\Delta\epsilon}{25\ \mathrm{kHz}}\right)$$

$5\sigma$ test of quantum gravity within $\mathcal{O}(1)$ day

+ time it takes to build the experiment…

# Conclusions

1. Holographic Quantum Gravity implies broadband stochastic distance fluctuations

2. Conventional homodyne readout of interferometers is sub-optimal to detect this signal

3. Photon counting readout ignores phase information, which yields a quantum advantage

4. If classical noise is mitigated to below the quantum noise, GQuEST can provide a test of quantum gravity within days

# FIN