Experimental constraints on quantum gravity fluctuations
&
Introduction to photon counting interferometry

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GQuEST

Outline

1. Constraints on Quantum Gravity Fluctuations (Pixellon Model)
    a) Precision Interferometers present and future
    b) Quantum Gravity constraints from interferometry
    c) Quantum Gravity constraints from astronomical image blurring
2. Basics of Interferometry:
    a) Homodyne Readout
    b) Photon Counting Readout

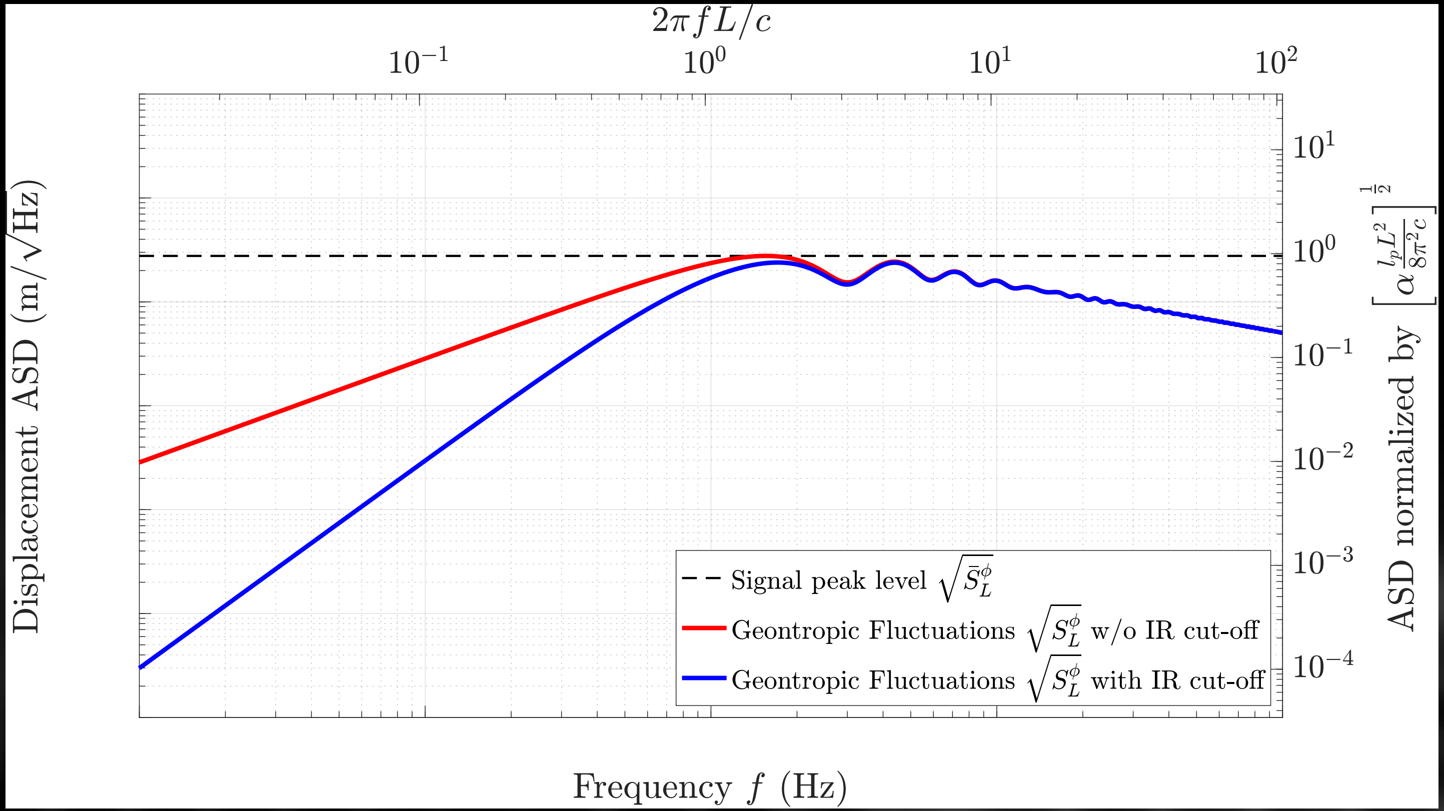
Quantum Gravity fluctuations: Pixellon Model

- Associate a stochastic scalar field to modular energy fluctuations: $\phi(\vec{x}, t)$
- The field gravitates, perturbing the metric: $ds^2 = -dt^2 + (1-\phi)\left(dr^2 + r^2 d\Omega^2\right)$

→IFO signal spectrum:

$$\sqrt{S^{\mathrm{sig}}(f)}$$

$$= \sqrt{\int_{-\infty}^{\infty} \left\langle \frac{\Delta L(\tau)}{L} \frac{\Delta L(0)}{L} \right\rangle e^{-2\pi i f \tau} d\tau}$$



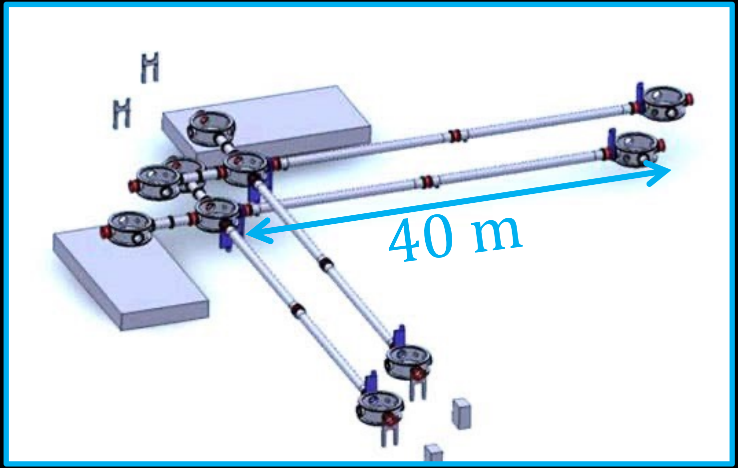
Li et al. (2023), SMV et al. (in prep.)

Existing Interferometers

LIGO (USA)

GEO600 (MPI, Germany)

Holometer (Fermilab)

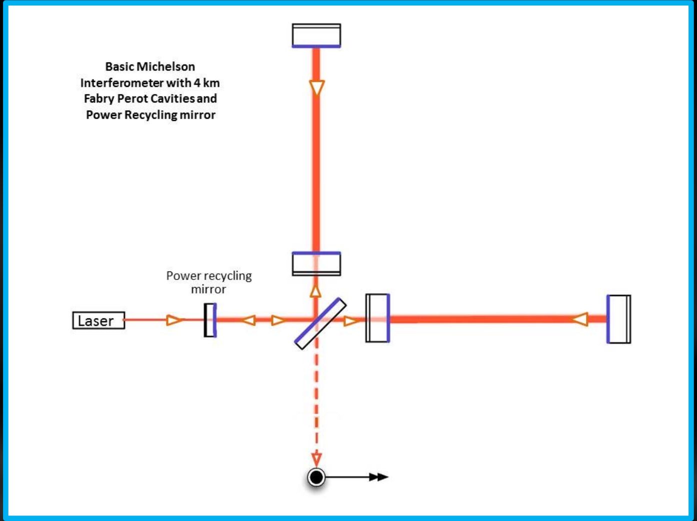


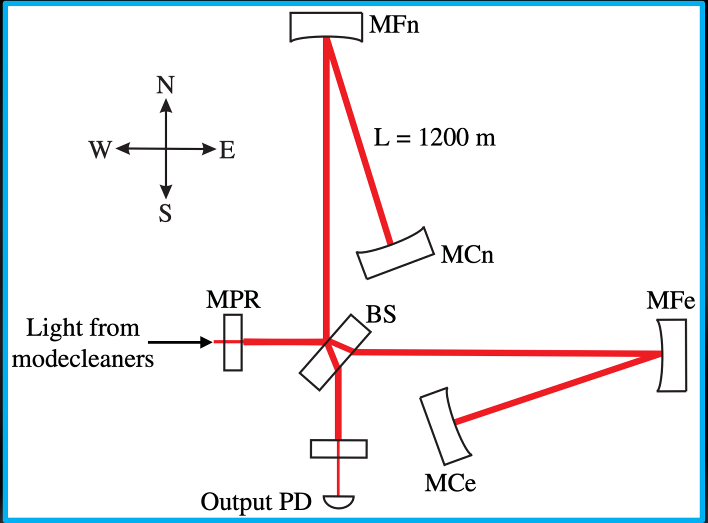
Sensitivity:

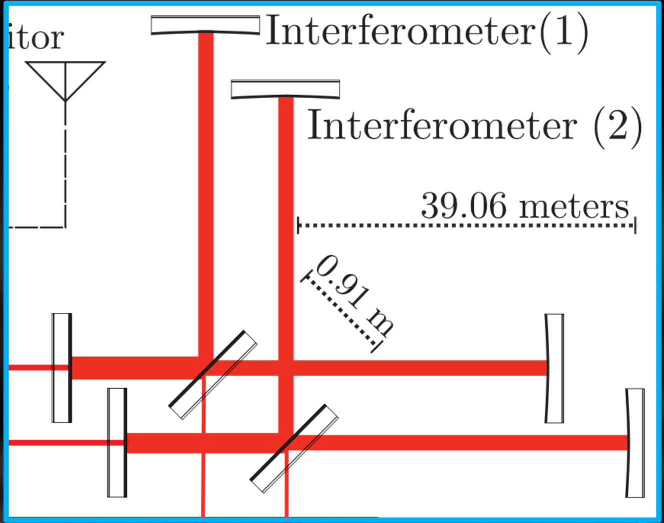
LIGO: $\delta L \approx 10^{-20}$ m @ 100 Hz

GEO600: $\delta L \approx 10^{-19}$ m @ 1000 Hz

Holometer: $\delta L \approx 10^{-18}$ @ 10 MHz







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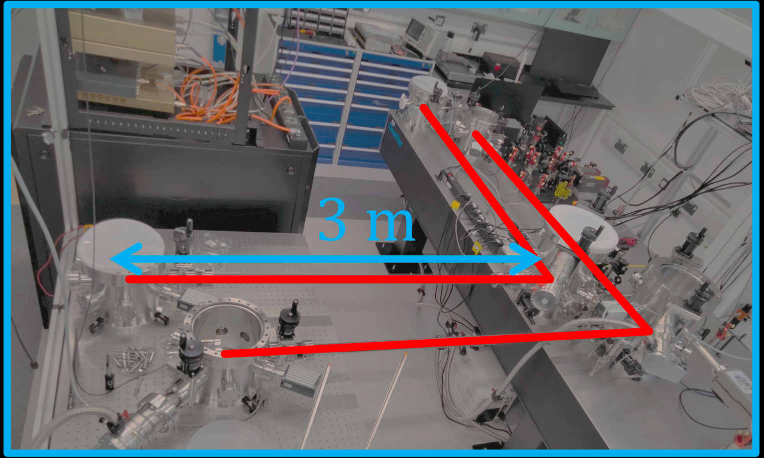
Interferometers being built:

QUEST (Cardiff University)



Sensitivity to length changes:

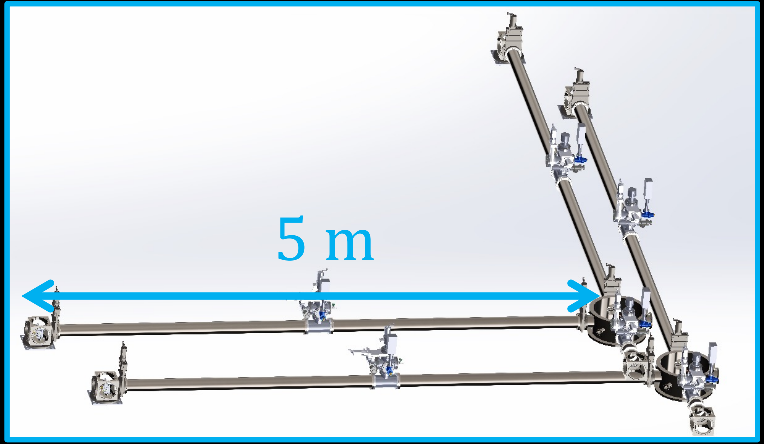
$\delta L \approx 10^{-19}$ m

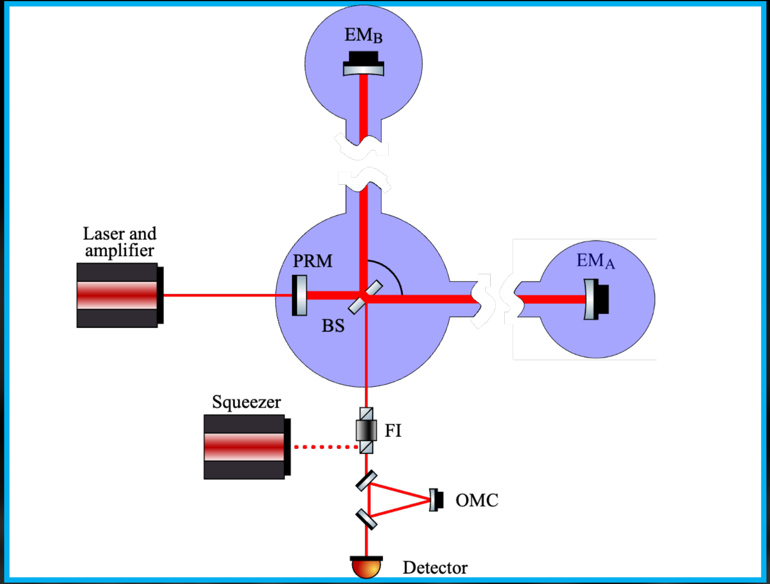
@ 10 MHz

$\delta L \approx 10^{-21}$ m

@ 10 MHz

GQuEST (Caltech)

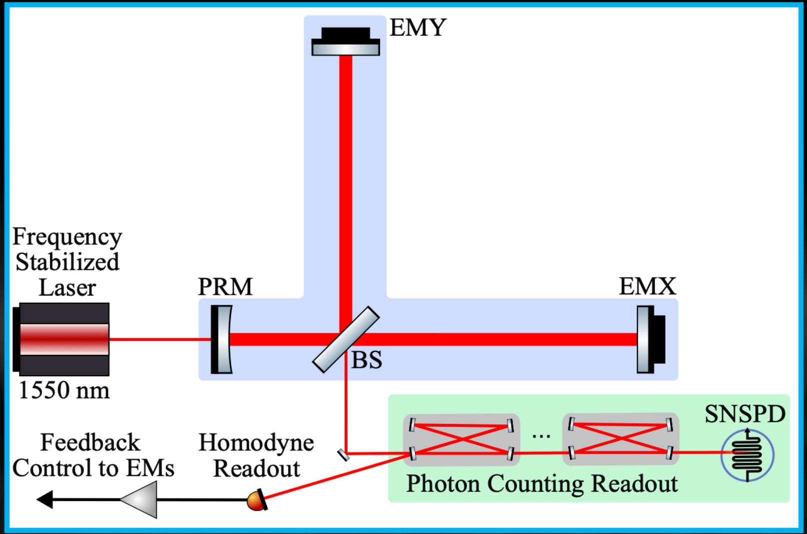




Detection statistic:

$$\propto \left[\frac{S_{\mathrm{Signal}}}{S_{\mathrm{Noise}}}\right]^2$$

$$\propto \frac{S_{\mathrm{Signal}}}{S_{\mathrm{Noise}}}$$



$$\frac{S_{\mathrm{signal}}}{S_{\mathrm{noise}}} \ll 1$$

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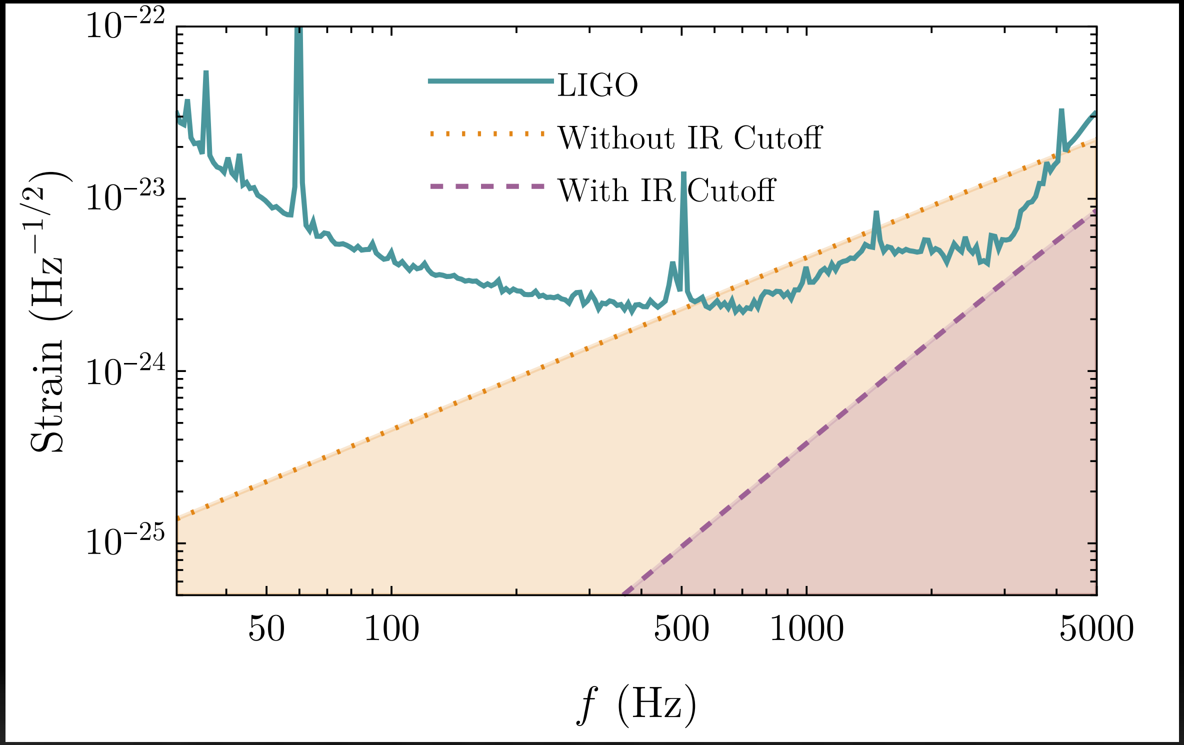
Pixellon Model: interferometric constraints

$$\max\left(\sqrt{S_L^{\text{sig}}(f)}\right) = \sqrt{\alpha\,\frac{l_p L^2}{8\pi^2 c}}$$

LIGO

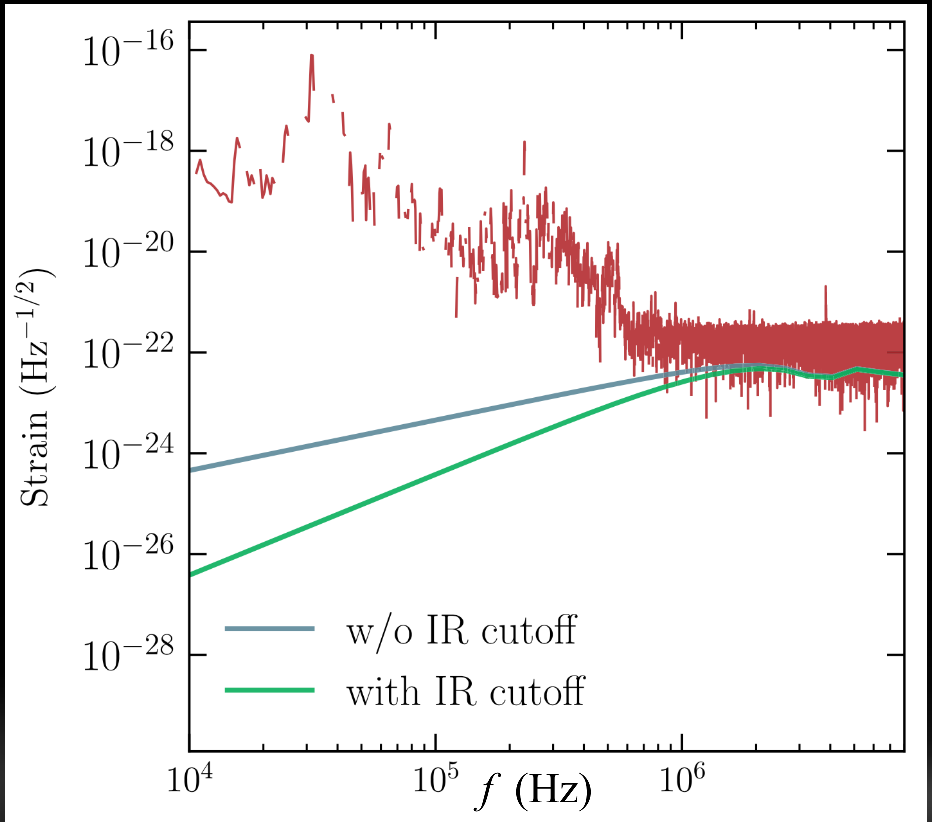


Bub et al. (2023)

$\alpha \lesssim 3$ (w/ IR cut-off)

$\alpha \lesssim 0.1$ (w/o IR cut-off)

Holometer

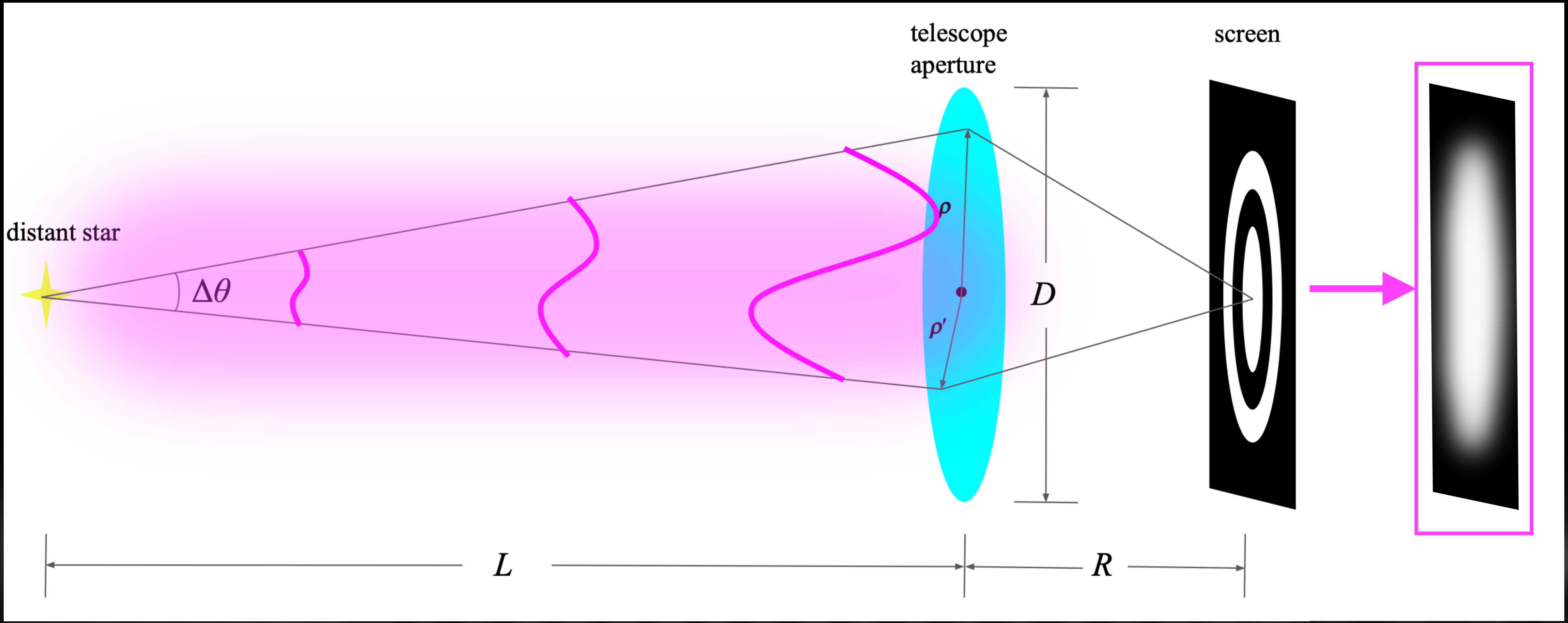


Li et al. (2023)

$\alpha \lesssim 0.7$ (w/ IR cut-off)

$\alpha \lesssim 0.6$ (w/o IR cut-off)

Quantum gravity fluctuations: constraints from astronomical image blurring



Lee et al. (2023)

See also Ng, Van Dam, Christiansen, Perlman, Steinbring, et al. (2006-2023)

Quantum gravity fluctuations: constraints from astronomical image blurring

“ … image distortion effects in the pixellon model are strongly suppressed […], thus evading all existing and future constraints. ”

Lee et al. (2023)

→ No blurring expected

“[The pixellon model is] shown to predict excessive blurring of images from distant sources.”

Hogan et al. (2023)

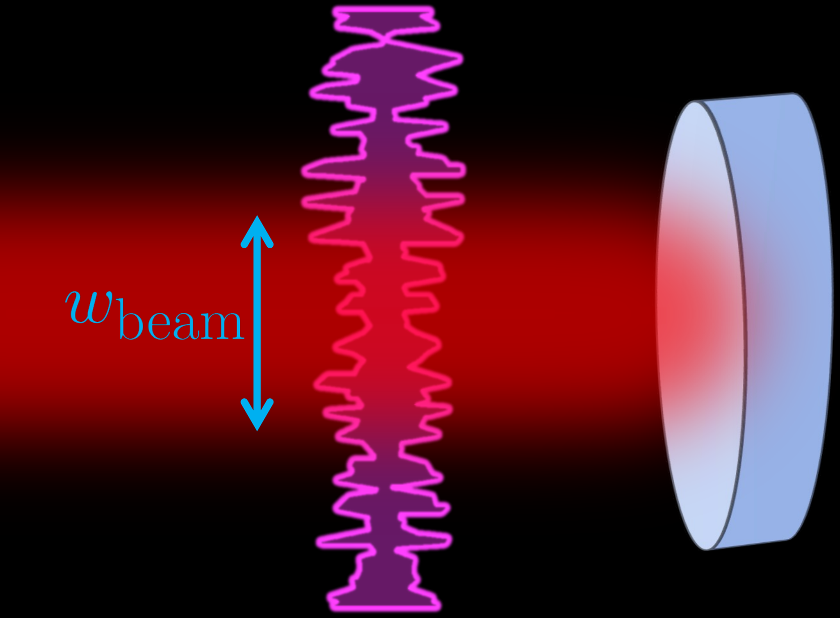
→ Too much blurring expected

“[Holographic quantum gravity] foam-induced blurring is described, analogous to atmospheric seeing from the ground. When scaled within the fields of view for the Fermi and Swift instruments, it fits all […] data […] in agreement with a holographic QG-favored formulation.”

Steinbring (2023)

→ Right amount of blurring observed

Correlation length of the fluctuations is crucial to observability

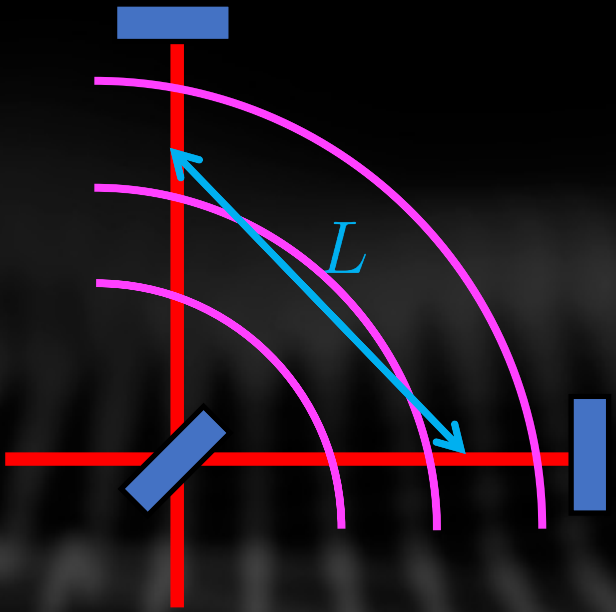


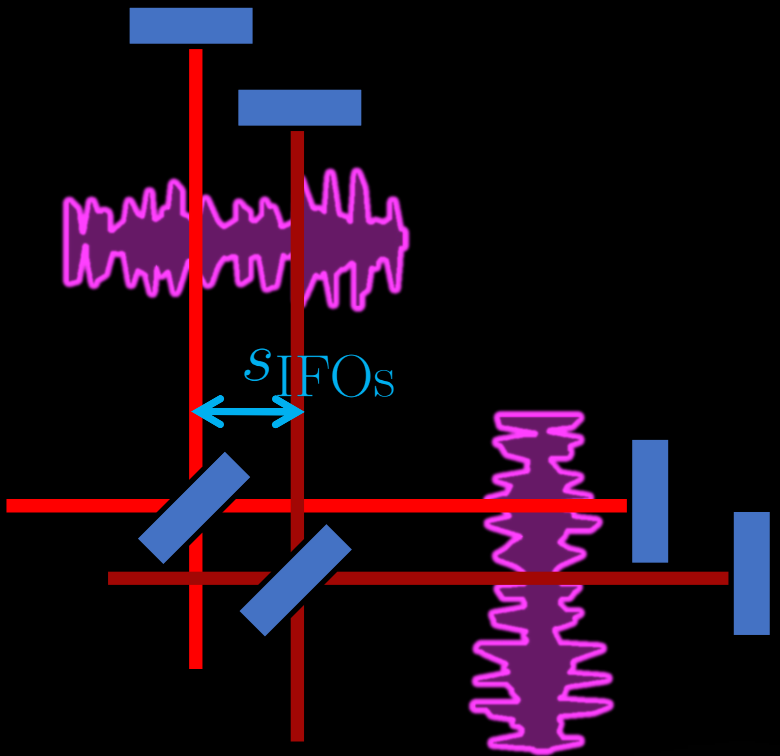
$l_{\mathrm{corr}} \ll w_{\mathrm{beam}}$

→ No signal

$l_{\mathrm{corr}} \gg L$

→ No signal





$l_{\mathrm{corr}} \ll s_{\mathrm{IFOs}}$

→ No correlated signal

Formalism to extract EM signatures from space-time fluctuations

- Perturbed Metric:

$$\begin{pmatrix} w_{\mathrm{T}}(\boldsymbol{r}) & 0 & 0 & 0 \\ 0 & w_{\mathrm{S}}(\boldsymbol{r}) & 0 & 0 \\ 0 & 0 & w_{\mathrm{S}}(\boldsymbol{r}) & 0 \\ 0 & 0 & 0 & w_{\mathrm{S}}(\boldsymbol{r}) \end{pmatrix}$$

- Two-point correlation functions:

$$\overline{w_{\mathrm{S}}(x,y,z)w_{\mathrm{S}}(x+\delta_x,y+\delta_y,z+\delta_z)} = \Gamma_{\mathrm{S}}\,\rho(\delta_x,\delta_y,\delta_z)$$

$$\rho(\delta_x,\delta_y,\delta_z) = \exp\left(-\sum_{i=x,y,z}\frac{\delta_i^2}{2\ell_i^2}\right),$$

OPEN ACCESS
IOP Publishing
Classical and Quantum Gravity
Class. Quantum Grav. 41 (2024) 075003 (31pp)
https://doi.org/10.1088/1361-6382/ad2970

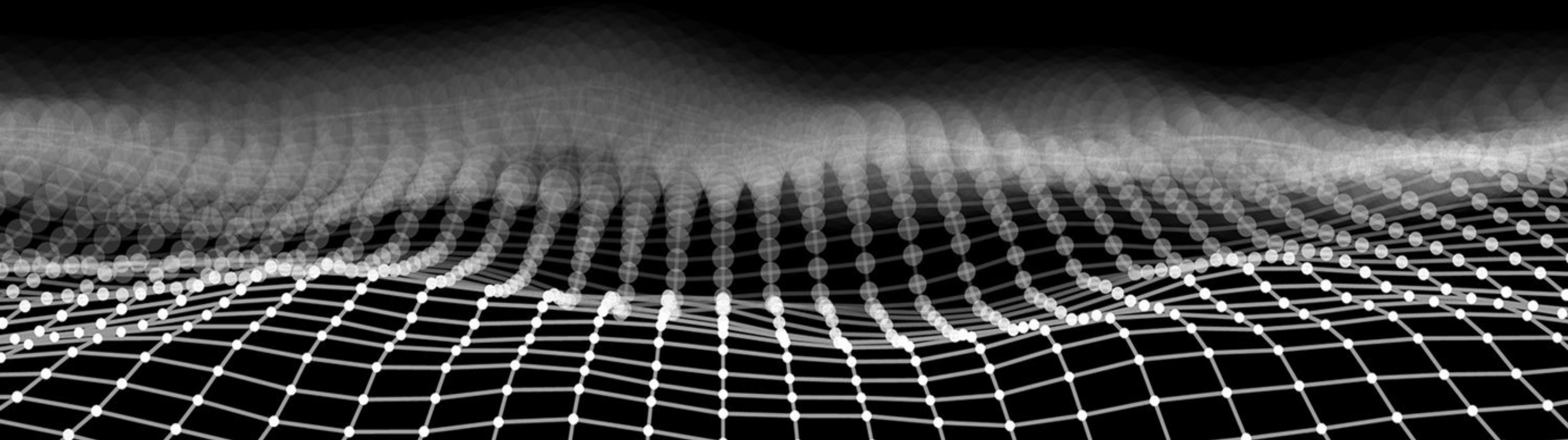
Extracting electromagnetic signatures of spacetime fluctuations

B Sharmila[1,*], Sander M Vermeulen[2,3]
and Animesh Datta[1]

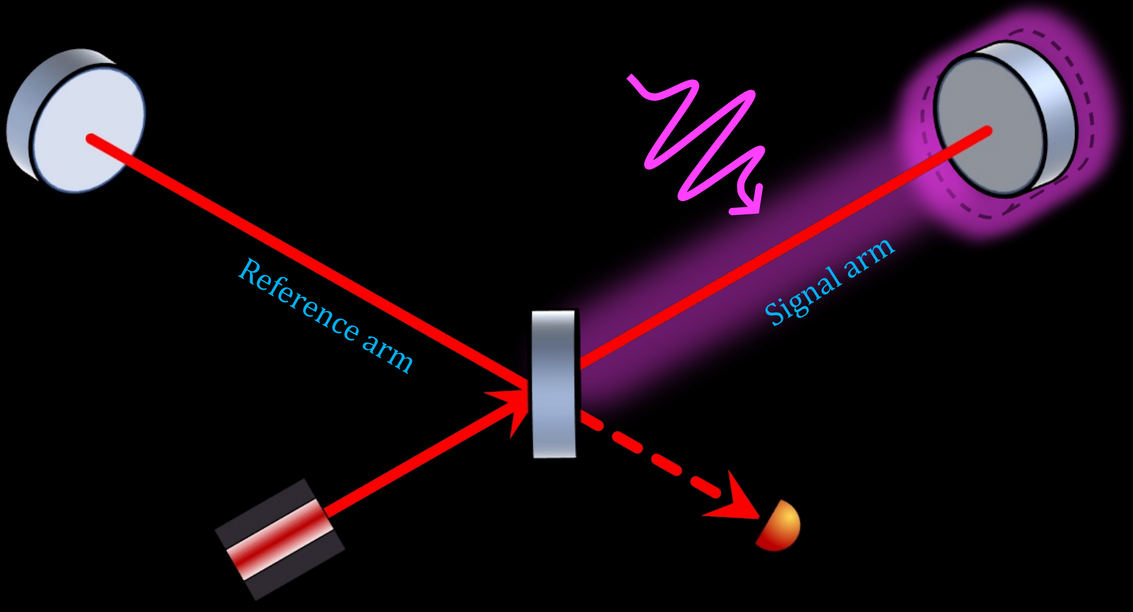
→ Solve relativistic EM wave equation on metric, get EM correlation tensor for experiment:

$$M^{m,m'}_{c;c'}(\{\boldsymbol{r}\};\{\boldsymbol{r'}\}) = E_{c_1}(\boldsymbol{r}_1)E_{c_2}(\boldsymbol{r}_2)\cdots E_{c_m}(\boldsymbol{r}_m)E^*_{c'_1}(\boldsymbol{r'}_1)E^*_{c'_2}(\boldsymbol{r'}_2)\cdots E^*_{c'_{m'}}(\boldsymbol{r'}_{m'})$$

Laser Interferometry & Photon Counting



Laser interferometry: measuring phase modulations



Perturbations $\Longrightarrow$ $\boxed{\delta L \quad \text{or} \quad \delta\phi}$

Modulates the power at the output

Phase modulation of the carrier field:

$$E_\mathrm{c}(t) = |E_\mathrm{c}|e^{i\omega_\mathrm{c}t} \longrightarrow E(t) = \underbrace{|E_\mathrm{c}|e^{i(\omega_\mathrm{c}t+\boxed{\delta\phi(t)})}}_{\text{Field in signal arm}}$$

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Sideband fields

$$E(t) = |E_c| e^{i(\omega_c t + \delta\phi(t))}$$

$$\boxed{\delta\phi(t) = \Phi \sin(\Omega t)}$$

Expansion for $\Phi \ll 1$

$$E(t) = |E_c| e^{i(\omega_c t + \Phi \sin(\Omega t))} \approx |E_c| e^{i\omega_c t} \left(1 + \frac{i}{2}\Phi \left[e^{-i\Omega t} + e^{+i\Omega t}\right]\right)$$

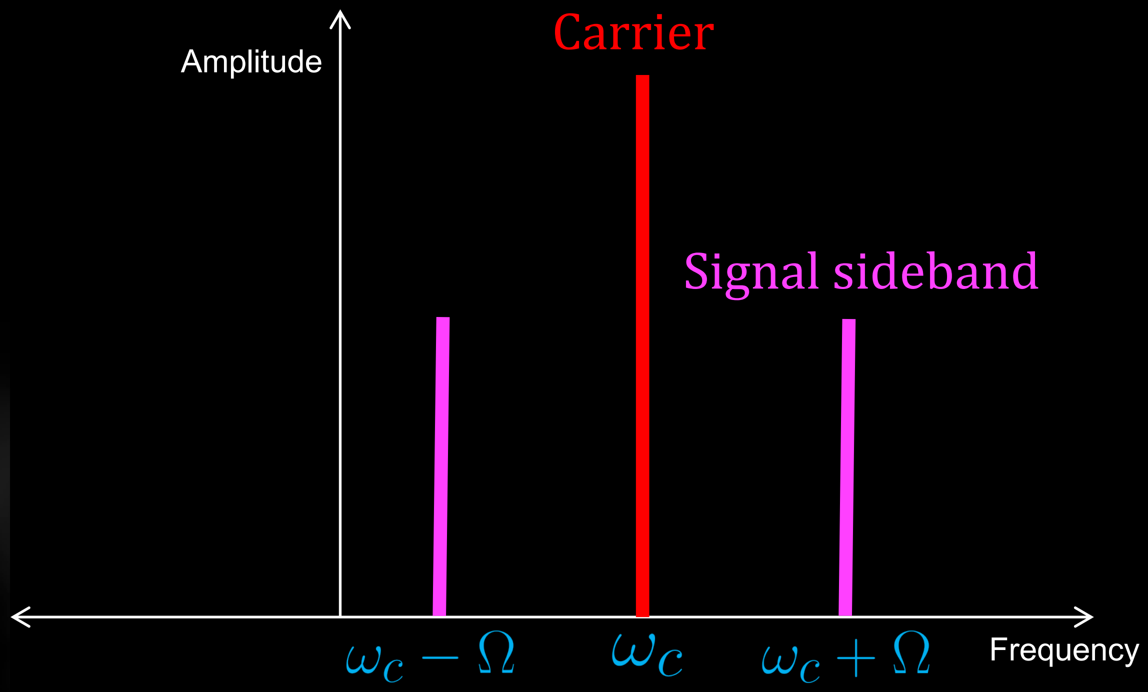
$$\rightarrow E(t) \approx E_c + E_{sig}$$

Carrier field:

$$E_c = |E_c| e^{i\omega_c t}$$

+ Signal sideband fields

$$E_{sig} = \frac{1}{2}|E_c|\Phi \left[e^{i(\omega_c - \Omega)t} + e^{i(\omega_c + \Omega)t}\right]$$



Sideband fields

$$E(t) = |E_c| e^{i(\omega_c t + \delta\phi(t))} \qquad \delta\phi(t) \equiv \boxed{A_{\text{sig}}(t)} \sim \mathcal{F}^{-1}\left\{ \sqrt{S_\phi^{\text{sig}}(f)} \right\} e^{i\eta_{\text{rand.}}(t)}$$

Expand for $A_{\text{sig}} \ll 1$

$$E(t) = |E_c| e^{i(\omega_c t + A_{\text{sig}}(t))}$$
$$\approx |E_c| e^{i\omega_c t}\left(1 + iA_{\text{sig}}\right)$$

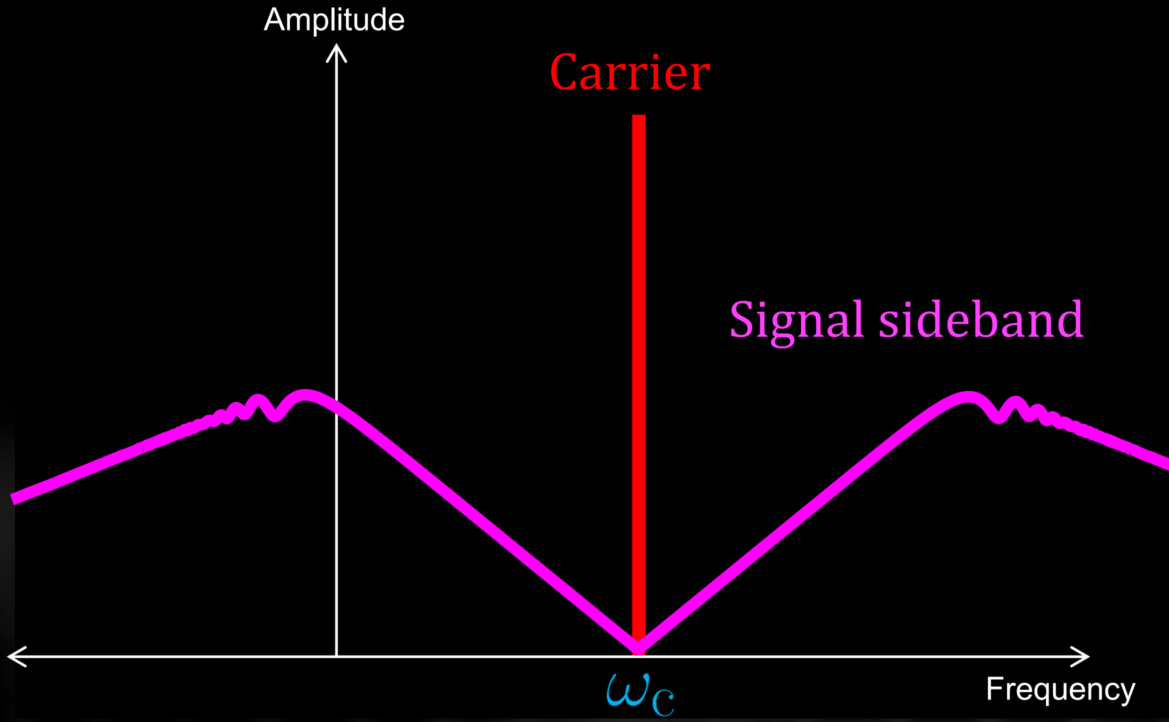
→ $E(t) \approx E_c + E_{\text{sig}}$

Carrier field:

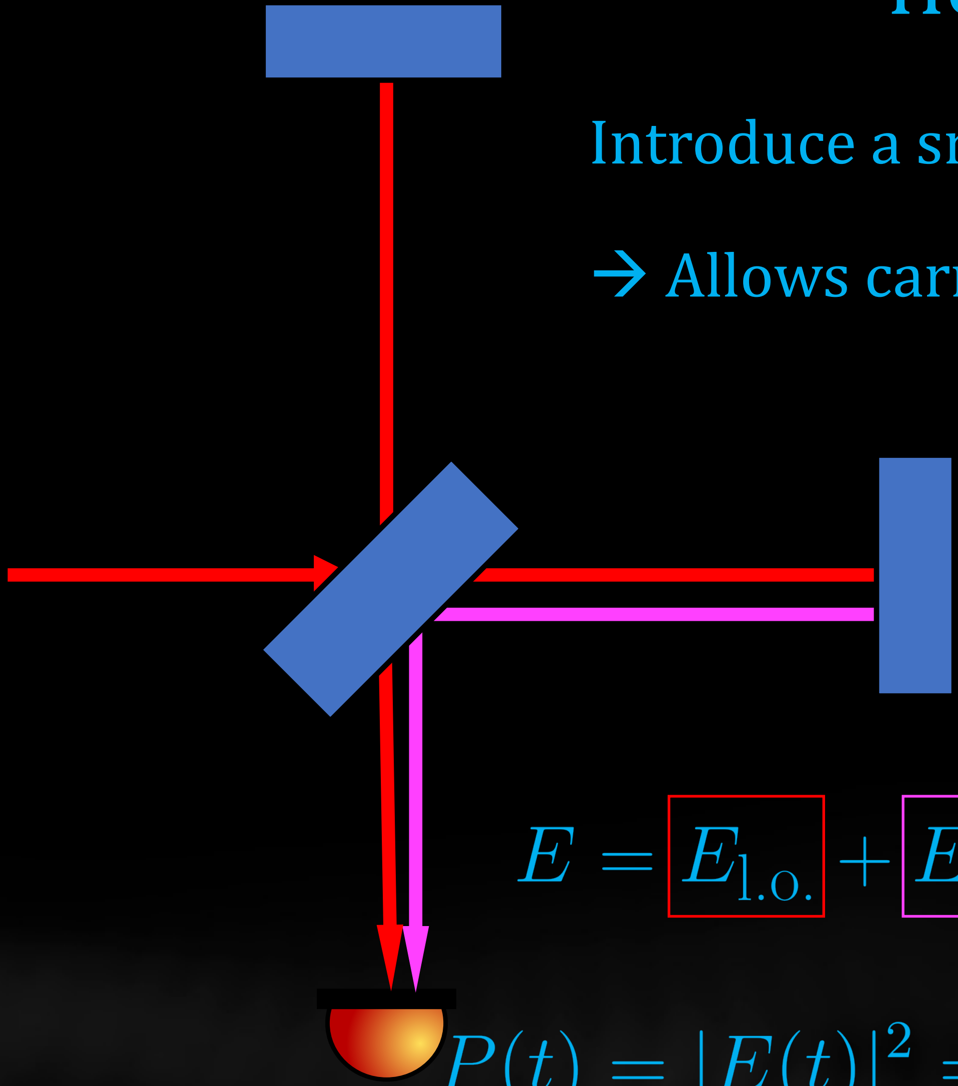
$$E_c = |E_c| e^{i\omega_c t}$$

+ Signal sideband fields

$$E_{\text{sig}} = E_c A_{\text{sig}}$$

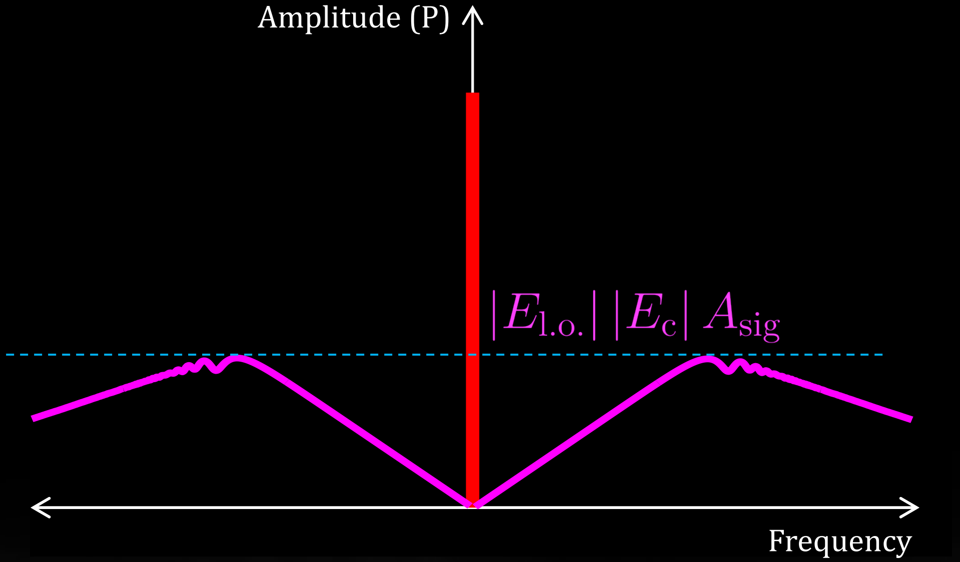


Homodyne Readout



Introduce a small static arm-length difference

→ Allows carrier field to leak into the output: $E_{\mathrm{l.o.}} = |E_{\mathrm{l.o.}}| e^{i\omega_c t}$



$$E = \boxed{E_{\mathrm{l.o.}}} + \boxed{E_{\mathrm{sig}}}$$

$$P(t) = |E(t)|^2 = \boxed{|E_{\mathrm{l.o.}}|^2} + \underbrace{\boxed{2|E_{\mathrm{l.o.}}||E_c|A_{\mathrm{sig}}}}_{\text{Sidebands beat with ‘local oscillator’}} + \boxed{|E_c|^2 A_{\mathrm{sig}}^2}$$

Quantum Shot Noise

Heisenberg uncertainty for coherent optical state:

$$\boxed{E = \sqrt{P}}$$

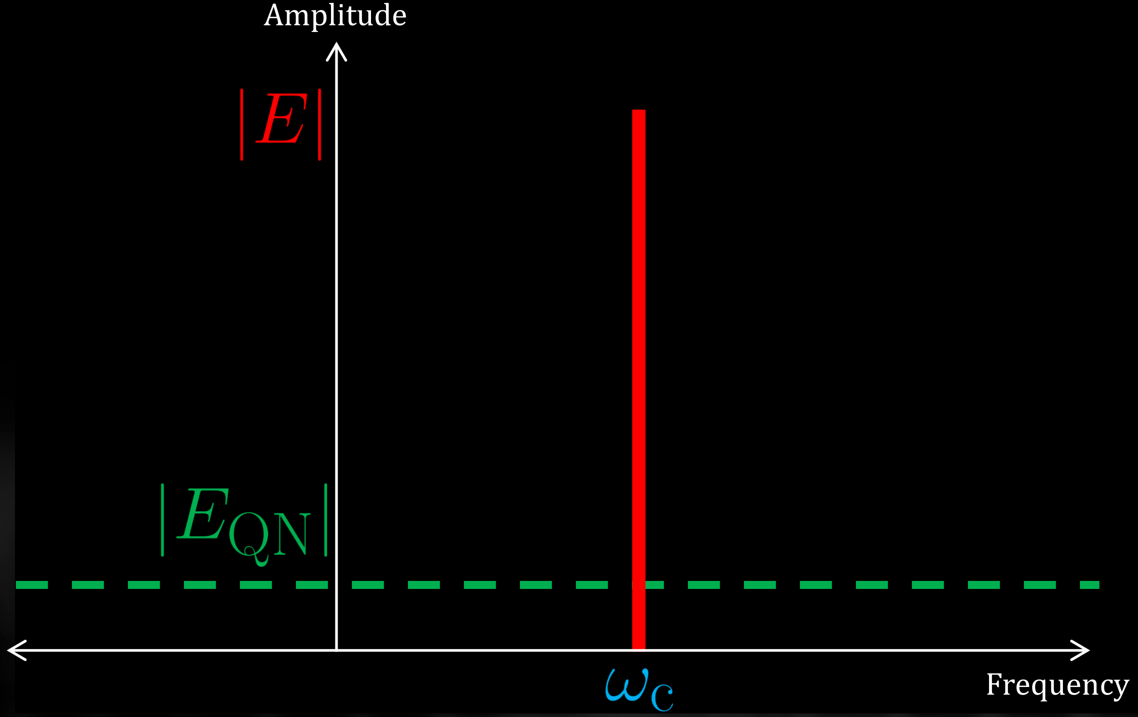
$$\left.\begin{array}{c} \sigma_N \cdot \sigma_\phi = 1 \\ \sigma_N = \sqrt{N} = \sqrt{\dfrac{|E|^2}{\hbar\omega}} \end{array}\right\} \sigma_\phi = \sqrt{\frac{\hbar\omega}{|E|^2}} = \sqrt{2S_\phi^{\mathrm{QN}}}$$

$(t_{\mathrm{meas.}} = 1\ \mathrm{s})$

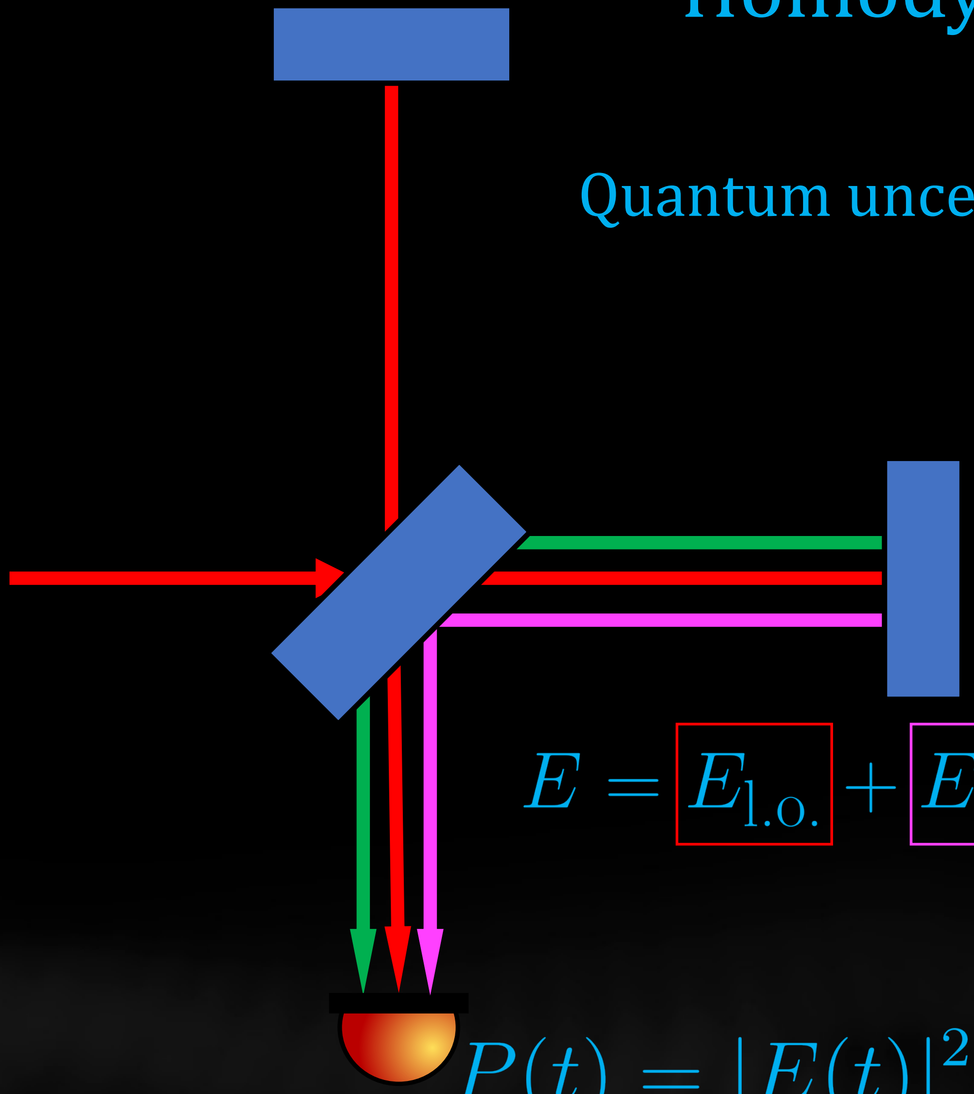
$$E(t) = |E_\mathrm{c}|e^{i\omega_\mathrm{c}t} \boxed{+\,\sigma_\phi}$$

Quantum noise ‘sidebands’:

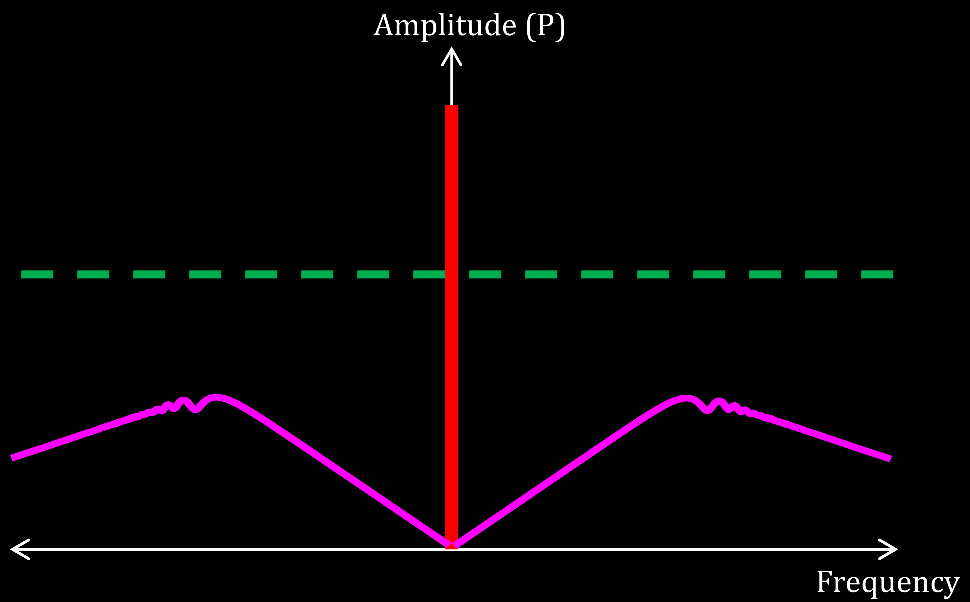
$$\boxed{E_{\mathrm{QN}} = \sqrt{\frac{\hbar\omega_c}{2}} \sum_{\omega=-\infty}^{\infty} \left[\hat{a}^\dagger e^{i(\omega_\mathrm{c}+\omega)t} + \hat{a}e^{i(\omega_\mathrm{c}-\omega)t}\right]}$$



Homodyne Readout: Shot Noise



Quantum uncertainty produces measured shot noise



$$E = \boxed{E_{\mathrm{l.o.}}} + \boxed{E_{\mathrm{sig}}} + \boxed{E_{\mathrm{QN}}}$$

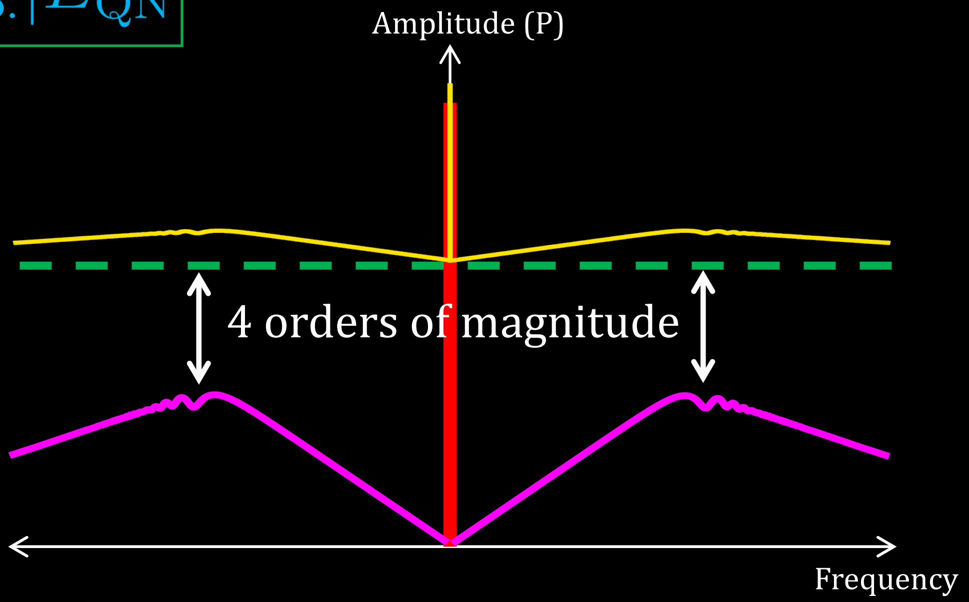
$$P(t) = |E(t)|^2 = \boxed{|E_{\mathrm{l.o.}}|^2} + \boxed{2|E_{\mathrm{l.o.}}||E_{\mathrm{c}}|A_{\mathrm{sig}}} + \boxed{2|E_{\mathrm{l.o.}}|E_{\mathrm{QN}}}$$

$$+\,\text{smaller terms}$$

Homodyne Readout: Statistics

$$P(t) = \boxed{|E(t)|^2} = \boxed{|E_{\text{l.o.}}|^2} + \boxed{2|E_{\text{l.o.}}||E_{\text{c}}|A_{\text{sig}}} + \boxed{2|E_{\text{l.o.}}|E_{\text{QN}}}$$

$$+ \text{smaller terms}$$



Detection statistic:

$$\chi^2 = \int \frac{\left(S_{\text{meas}}(f) - S_{\text{noise}}\right)^2}{\text{Var}\left(S_{\text{meas}}\right)} df \propto \left(\frac{S_{\text{signal}}}{S_{\text{noise}}}\right)^2$$

Can we do better?

→Yes, with photon counting!

Photon Counting: Intuition

- Homodyne readout measures time-dependence, i.e. phase/frequency of the signal

- The signal model does not specify these properties…

→ time-dependence/phase/frequency info is useless for finding a signal that is stationary/stochastic/broadband

→ Devise a quantum measurement that does not provide useless info, in exchange for useful info

Photon Counting

Measure the number of photons exactly:

$$\left.\begin{aligned} \sigma_N \cdot \sigma_\phi &= 1 \\ \sigma_N &\to 0 \end{aligned}\right\} \implies \sigma_\phi \to \infty$$

→No phase info measured

→Maximum info on signal power

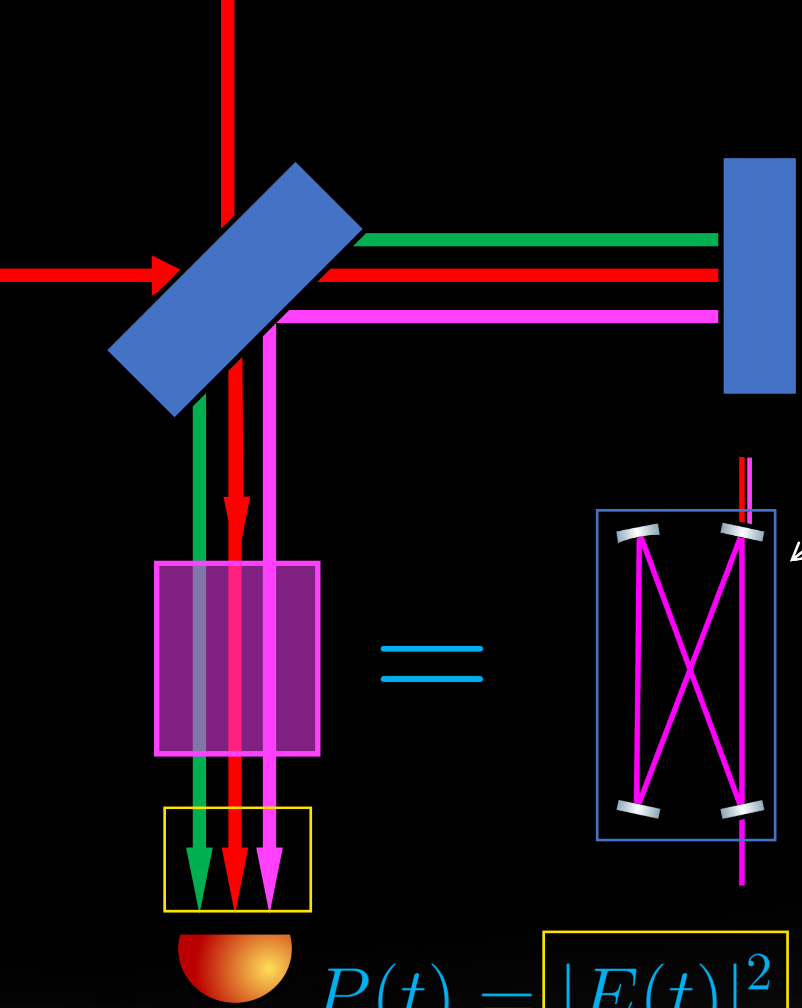
Practical challenges with this approach:

- Too much light to discern single signal photons
- Too many non-signal photons

→ Can't count photons precisely

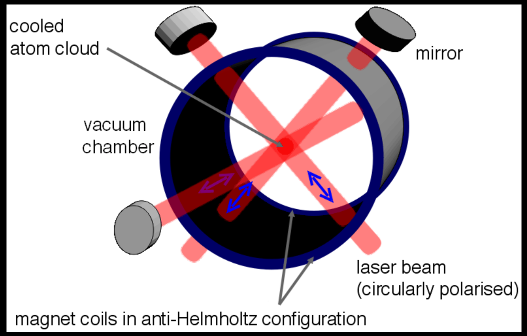
$$\sigma_N \neq 0$$

Photon Counting: Filtering



Narrowband optical filter

or



or

?

$$P(t) = \boxed{|E(t)|^2}$$

$$= \cancel{\underbrace{|E_{\mathrm{l.o.}}|}_{\rightarrow 0}\left(|E_{\mathrm{l.o.}}| + |E_{\mathrm{c}}|A_{\mathrm{sig}} + E_{\mathrm{QN}}\right) +} \boxed{|E_{\mathrm{c}}|^2 A_{\mathrm{sig}}^2} + \boxed{2|E_{\mathrm{c}}|A_{\mathrm{sig}}E_{\mathrm{QN}}}$$

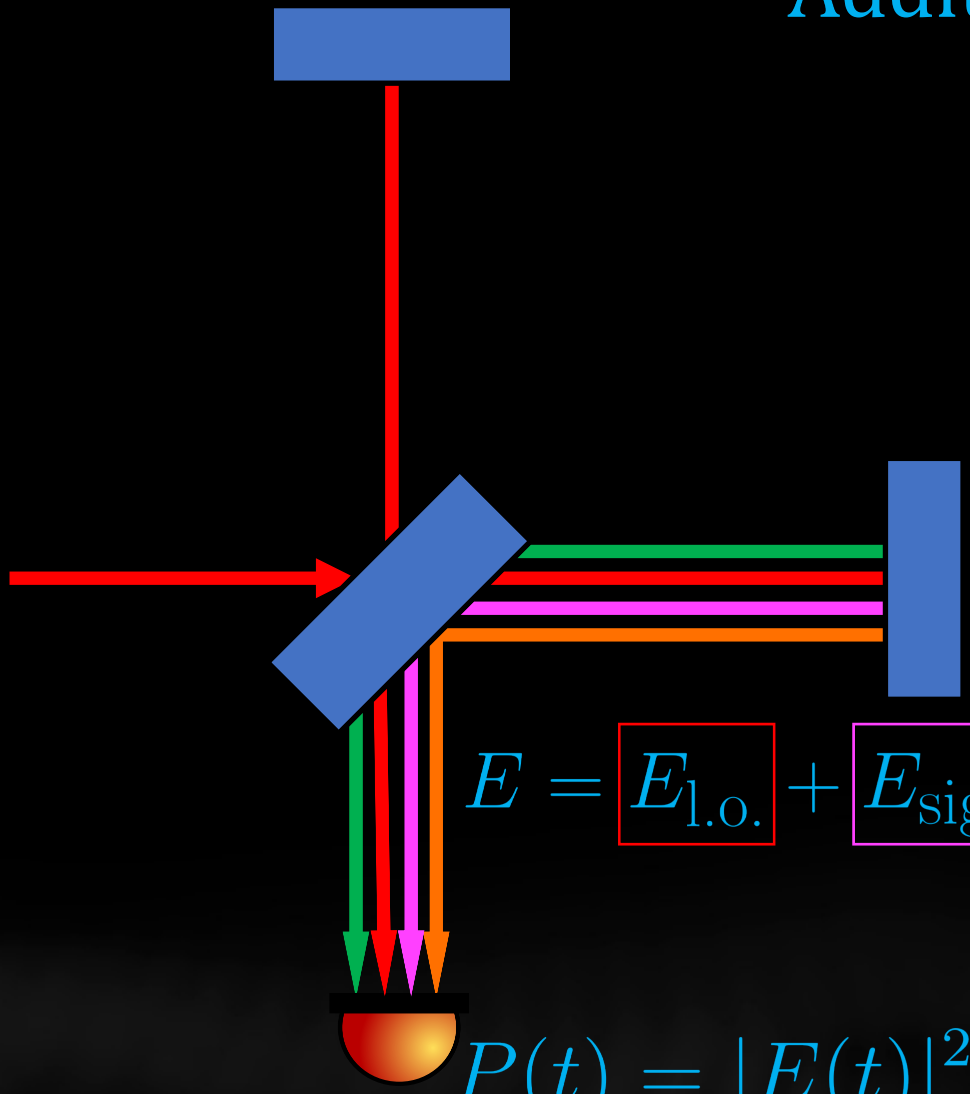
Photon Counting: Statistics

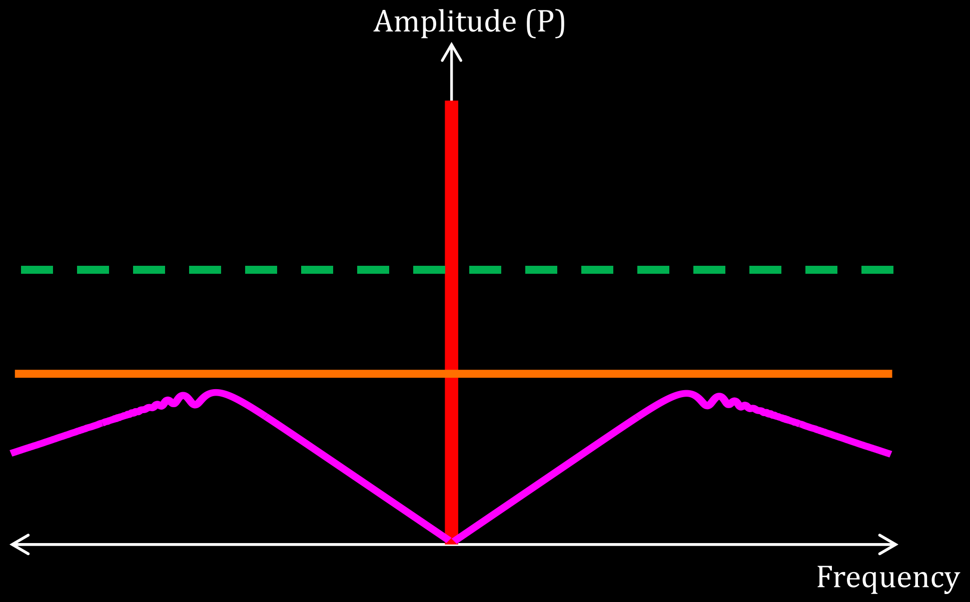
Detection statistic:

$$\left.\begin{aligned} \chi^2 &= \frac{N^2}{\mathrm{Var}\left(N\right)} = \frac{N^2}{N} = N \\ N &\propto \frac{\left| |E_\mathrm{c}|^2 A_\mathrm{sig}^2 \right|}{\hbar\omega} = \frac{S_\phi^\mathrm{sig}}{2 S_\phi^\mathrm{QN}} \end{aligned}\right\} \quad \chi^2 \propto \frac{S_\mathrm{signal}}{S_\mathrm{noise}}$$

Recall for Homodyne readout: $\chi^2 \propto \left(\frac{S_\mathrm{signal}}{S_\mathrm{noise}}\right)^2$

Additional Classical Noise





$$E = \boxed{E_{\text{l.o.}}} + \boxed{E_{\text{sig}}} + \boxed{E_{\text{QN}}} + \boxed{E^{\text{Cl.}}_{\text{Noise}}}$$

$$P(t) = |E(t)|^2 = \boxed{|E_{\text{l.o.}}|^2} + \boxed{2|E_{\text{l.o.}}||E_{\text{c}}|A_{\text{sig}}} + \boxed{2|E_{\text{l.o.}}|E_{\text{QN}}}$$
$$+ \boxed{2|E_{\text{l.o.}}||E_{\text{c}}|A^{\text{Cl.}}_{\text{Noise}} + |E_{\text{c}}|^2\left(A^{\text{Cl.}}_{\text{Noise}}\right)^2}$$

Photon Counting: Statistics

Homodyne readout: $$\chi^2 \propto \left(\frac{S_{\text{signal}}}{S_{\text{noise}}}\right)^2$$

Photon counting w/o classical noise: $$\chi^2 \propto \frac{S_{\text{signal}}}{S_{\text{noise}}}$$

Photon counting w/ classical noise: $$\boxed{\chi^2 \propto \frac{\left(S_{\text{signal}}\right)^2}{\left(S^{\text{Qu.}}_{\text{noise}}\right)\left(S^{\text{Cl.}}_{\text{noise}}\right)}}$$

Conclusions

1. Current experiments place a constraint on the Pixellon model $\alpha \lesssim O(1)$
2. Two-point correlations of quantum gravity fluctuations are an integral part of testable predictions
3. Photon counting interferometry is fundamentally more sensitive than conventional interferometry
4. GQuEST can test the Pixellon Model with $\alpha = 1$ within $O(1)$ day of measurement time (at 5σ)

FIN